GM-PHD Filter Multi-target Tracking in Sonar Images

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ABSTRACT
The Gaussian Mixture Probability Hypothesis Density (GM-PHD) Multi-target Tracker was developed as an extension to the GM-PHD filter to provide track continuity. The algorithm is demonstrated on forward-looking sonar data with clutter and is compared with the results from the Particle PHD filter.

1. INTRODUCTION
The Probability Hypothesis Density (PHD) filter,¹ or the first-order moment of a multiple-target posterior distribution, was developed to provide a computationally feasible alternative to propagating a multiple-target distribution. The Sequential Monte Carlo implementation of the PHD filter, called the Particle PHD Filter, was developed to provide a practical solution to the PHD recursion² using a particle filter approach. The set of target states and the time-varying number of targets are estimated at each time-step. The Particle PHD filter provides target state estimates at each iteration but track continuity is not maintained. Techniques have been developed to enable track continuity for the Particle PHD filter³⁴⁵. A practical implementation of the Particle PHD filter with track continuity has been demonstrated on forward-looking sonar,⁶ which showed that the technique gives comparable performance to conventional techniques for multiple-target tracking with Kalman filters in low levels of clutter.

The Gaussian Mixture Probability Hypothesis Density (GM-PHD) Filter⁷ provides a closed form solution to the PHD filter. Under linear/Gaussian assumptions, it was shown that when the prior intensity function is a Gaussian mixture, the updated posterior intensity function is also a Gaussian mixture. The PHD or intensity function is given by a weighted sum of Gaussians. The number of Gaussians in the mixture is managed by pruning, to eliminate Gaussian components with low weights, and merging, to combine components with similar means. The initial implementation of the GM-PHD filter provided estimates for the set of target states at each point in time but did not ensure continuity of the individual target tracks. The recently proposed GM-PHD Filter Multi-Target Tracker⁸ showed that the trajectories of the targets can be determined directly from the evolution of the Gaussian mixture and that single Gaussians within this mixture accurately track the correct targets. It was also shown to give better performance than the track-oriented MHT filter in high-clutter environments.

We demonstrate that the GM-PHD Filter Multi-target tracker can successfully track a time-varying number of obstacles in forward-looking sonar data in higher levels of clutter than has been shown for the Particle PHD filter.

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2. FORWARD-LOOKING SONAR

2.1. Sonar Specification

The sequences of images were obtained from a forward looking multi-beam sonar which was fitted to an Autonomous Underwater Vehicle (AUV). The vehicle was travelling at a rate of approximately 1 knot over a region with stationary targets on the seabed. The sonar was mounted on the front of the AUV scanning forwards for a range of 40m and was angled towards the seabed. The sonar scanned an angular region of 120 degrees, using 120 beams each with a vertical beam width of 1 degree and a horizontal beam width of 40 degrees, as shown in figure 1. The sonar had an operating frequency of 600 kHz.

2.2. Segmentation and Feature Extraction

Multi-beam sonar images can be very noisy, due to reverberation from the seabed, surface or water column. Median filtering is applied to reduce this affect. The objects which we wish to track have a higher reflectivity property than the surrounding environment, so the measurements are determined by thresholding the sonar images on intensity.

A double threshold is applied by firstly using adaptive threshold to identify regions of high reflectivity and then using a higher threshold to identify the regions with the highest returns. Figure 3 shows the sonar image, the image after filtering, and the segmented image from which the measurements are determined by taking the centroids of high intensity regions. The procedure used for tracking is shown in figure 2.

A recent study with the Particle PHD filter demonstrated an application of the algorithm with track continuity on forward-looking sonar data. A coarse segmentation was first used to identify regions of interest where the objects were likely to be followed by a more elaborate segmentation to obtain the measurements. This process helped reduce the number of false measurements from clutter used as input. In our approach used here, we simply use the double thresholding described above which results in higher clutter levels. We demonstrate that the GM-PHD filter copes well in these circumstances and compare the results with those obtained for the Particle PHD filter in lower clutter levels.
3. GM-PHD FILTER TRACKING

3.1. Tracking Model

The linear Gaussian dynamic model with following state space model is used,

\[
x_{t+1} = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} x_t + \begin{pmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{pmatrix} v_t,
\]

and observation model,

\[
z_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x_t + w_t.
\]

\(v_t\) and \(w_t\) are the process and measurement noises respectively, which are uncorrelated.

The state vector is defined as the 2D position and velocity vector of the target:

\[
x_t = \begin{pmatrix} x_t \\ y_t \\ \dot{x}_t \\ \dot{y}_t \end{pmatrix}^T.
\]

For simplicity, we write this as,

\[
x_{t+1} = F_t x_t + \Gamma_t v_t,
\]

\[
z_t = H_t x_t + w_t.
\]

The matrices \(F_t\), \(\Gamma_t\), and \(H_t\) are defined as the matrices used above. The random processes \(v_t\) and \(w_t\) represent the uncorrelated system and measurement noises which are determined from system covariance matrix \(Q_t\) and observation covariance matrix \(R_t\) respectively. The system covariance matrix reflects the uncertainty in the motion of the targets and the observation covariance matrix reflects errors in the measurements.
3.2. The Kalman Filter Equations

Each component in the Gaussian Mixture is predicted and updated using the Kalman filter equations. The predicted state estimate $m_{t-1}$ and state covariance to time $t$ are given by,

$$m_{t} = F_{t} m_{t-1}, \quad (6)$$

$$P_{t} = Q_{t} + F_{t} P_{t-1} F^{T}_{t}, \quad (7)$$

When measurement $z$ is received, the updated estimate $m_{t}$ and covariance $P_{t}$ are given by,

$$m_{t}(z) = m_{t} + K_{t} (z - H_{t} m_{t}), \quad (8)$$

$$P_{t} = (I - K_{t} H_{t}) P_{t}, \quad (9)$$

where $K_{t}$ is the Kalman gain.

3.3. The GM-PHD Filter Tracker

The implementation of the GM-PHD Multi-Target Tracker proceeds as follows.

**Initialisation**

Initialise the algorithm with the weighted sum of $J_0$ Gaussians,

$$D_{00} = \sum_{i=1}^{J_0} w_{0}^{(i)} N(x; m_{0}^{(i)}, P_{0}^{(i)}),$$

Each Gaussian in the mixture is given a label,

$$L_{0} = \{L_{0}^{(1)}, \ldots, L_{0}^{(J_0)}\}.$$ 

The sum of weights,

$$\sum_{i=1}^{J_0} w_{0}^{(i)} = \hat{T}_0,$$

is the expected number of targets at the start of the algorithm.

**Prediction**

In the prediction step, each Gaussian component is predicted with (6) and (7) to give,

$$D_{S,t|t-1}(x) = p_{S} \sum_{j=1}^{J_{t-1}} w_{t-1}^{(j)} N(x; m_{S,t|t-1}^{(j)}, P_{S,t|t-1}^{(j)}),$$

where $p_{S}$ is the probability of survival. In addition, new Gaussian components are added for the spontaneous birth model

$$\gamma_{t}(x) = \sum_{i=1}^{J_{t}} w_{t}^{(i)} N(x; m_{t}^{(i)}, P_{t}^{(i)}),$$
The intensity, $D_{t|t-1}$, to time $t$ is then

$$D_{t|t-1}(x) = D_{S,t|t-1}(x) + \gamma_t(x), \quad (17)$$

The set of labels from the previous time step are concatenated with new labels from the Gaussians introduced for the spontaneous birth model to form the set of prediction labels,

$$L_{t|t-1} = L_t \cup \{L_{t}^{(1)}, \ldots, L_{t}^{(J_t)}\}. \quad (18)$$

**Update**

When the measurements, $Z_t = \{z_{t,1}, \ldots, z_{t,m_t}\}$, are received at time $t$, compute the posterior intensity,

$$D_{t|t}(x) = (1 - p_D)D_{t|t-1}(x) + \sum_{z \in Z_t} \sum_{j=1}^{J_{t|t-1}} w_t^{(j)}(z) \mathcal{N}(x; m_t^{(j)}(z), P_t^{(j)}), \quad (19)$$

where the means, $m_t^{(j)}$, and covariances, $P_t^{(j)}$, are computed using (8) and (9), and the weights are calculated with the PHD filter update equation,$^7$

$$w_t^{(j)}(z) = \frac{p_D w_t^{(j)} \mathcal{N}(z; H_t m_t^{(j)} - H_t \bar{x}_t, R_t + H_t P_t^{(j)} H_t^T)}{\kappa_t(z) + p_D \sum_{l=1}^{J_{t|L}} w_t^{(l)} \mathcal{N}(z; H_t m_t^{(l)} - H_t \bar{x}_t, R_t + H_t P_t^{(l)} H_t^T)} \quad (20)$$

$P_D$ is the probability of detection, $\lambda_t$ is the expected number of clutter points and $c_t$ is the distribution of these across the state space.

There are $(1 + |Z_t|)J_{t|t-1}$ Gaussian components, $(1 + |Z_t|)$ for each prediction term. For each component, the same label as its related prediction component is assigned to form the set of update labels,

$$L_{t,u} = L_{t|t-1}^{D_{t|t-1}} \cup L_{t|t-1}^{L_{t|t}^{1}} \cup \ldots \cup L_{t|t-1}^{L_{t|t}^{c_t}}. \quad (21)$$

**Pruning**

The Gaussian components with low weights are eliminated in the pruning stage to ensure that the complexity of the algorithm does not grow exponentially. Let the weights $w_t^{(1)}, \ldots, w_t^{(N_t)}$ be those which are below the truncation threshold $T$, and let

$$D_t := \frac{\sum_{i=1}^{J_t} w_t^{(i)}}{\sum_{j=N_t+1}^{J_t} w_t^{(j)}} \sum_{i=N_t+1}^{J_t} w_t^{(i)} \mathcal{N}(x; m_t^{(i)}, P_t^{(i)}). \quad (22)$$

**Merging**

In the merging stage, Gaussian components whose distance between the means falls within a threshold, $U$, defined by the covariance matrix are merged. For example, if the means of components $i$ and $j$ are such that,

$$(m_t^{(i)} - m_t^{(j)})^T (P_t^{(i)})^{-1} (m_t^{(i)} - m_t^{(j)}) \leq U. \quad (23)$$
then merge. (The full procedure is given in\(^7,8\) ) If two or more components still have the same label \( L_t^{(i)} \), then this is given to the one with the largest weight \( \tilde{w}_t^{(i)} \) and new labels are assigned to the other components.

**State Estimation**

Target states are determined from Gaussians whose weights are above a specific threshold. In addition, the Gaussians that have previously had weights above this threshold are also taken to be target states. These are identified by their label, i.e. the set of live tracks from time \( t \) is

\[
\hat{L}_t = \{ L_t^{(i)} : \tilde{w}_t^{(i)} > 0.5 \} \tag{24}
\]

and the set of estimates is

\[
\hat{X}_t = \{ m_t^{(i)} : L_t^{(i)} \in \hat{L}_t, j = 1, \ldots r \}. \tag{25}
\]

### 4. RESULTS

Figures 4 and 5 show results of the Particle PHD and GM-PHD filters respectively. The first of these uses a more complex pre-processing procedure for determining the measurements to reduce clutter levels and the second uses simple thresholding which gives more clutter points. The average number of clutter points with the first method was less than 1 and with the second around 5.

While the empirical distribution from the Particle PHD filter can handle high clutter levels and estimate the correct number of targets, estimating the target states relies on clustering techniques which can lead to inaccurate and false estimates being obtained. The target states are determined from the GM-PHD filter by taking the Gaussians with the highest weights. In simulations, it has been shown that individual Gaussians can accurately track the correct targets\(^8\) in high clutter levels. This has a number of advantages over the particle implementation. First, the complexity of the algorithm is lower, the number of Gaussians used (maximum of 200 after pruning and merging compared with 1000 particles per target). Second, the means of the Gaussians are known and don’t need to be determined through clustering techniques. Finally, individual Gaussians determine the target states and are tracked more reliably through the labelling process compared to track continuity techniques developed for the Particle PHD filter. For these reasons, the GM-PHD filter can perform better under higher clutter levels.

The number of clutter points in the sequence ranged between 0 and 12 points. Figure 6 gives an example of the original, filtered and segmented images used. There are 5 false alarms in this example.
5. CONCLUSIONS

The GM-PHD filter Multi-target Tracker has been implemented for tracking obstacles in forward-looking sonar data. It is shown that the algorithm can successfully track the trajectories of a variable number of targets in a high clutter environment and estimate the correct number of targets. The performance of the algorithm has been compared with the Particle PHD filter and gives comparable tracking performance in higher clutter levels.

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