Digital Signal Processing

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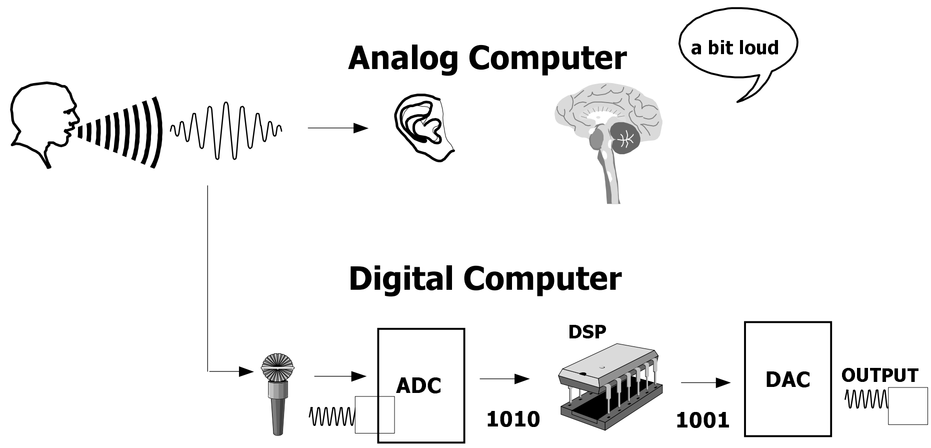
Section Contents

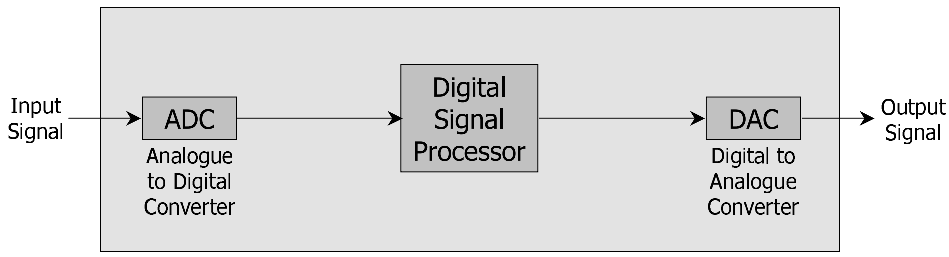
- Signal processing? … Was is das?
- Digital signal processors
- Analog to digital signal conversion
- Basic concepts
- Summary

What Is DSP?



What is DSP?

- **Digital Signal Processing** – the processing or manipulation of signals using digital techniques



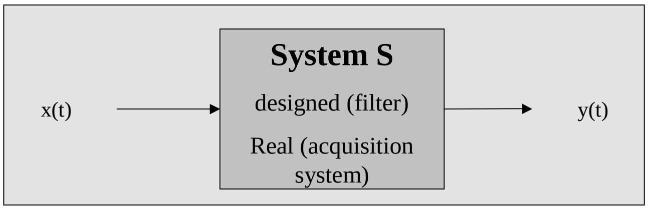
What is DSP Used For?

…And much more!

Section Contents

- Continuous Linear Time Invariant (LTI) Systems
- Digitizing the signal
- Sampling and aliasing
- Sampling and Frequency effects
- Summary

Continuous Linear Time Invariant Systems



- Time Invariance (also called stationarity)
  - The system does not evolve through time: same input means same output
  - Analysis done at a certain time is valid at another
- Linearity: $S[ax_1(t) + bx_2(t)] = aS[x_1(t)] + bS[x_2(t)]$

Causality & Stability

<u>Causality</u>

A LTI system is said to be causal if its output at any given time depends only on the input values up to and including that time.

Obvious for any practical system (we cannot predict the future)

Some desirable functions (ideal differentiator) require non-causal functions. Any non-causal function is not realisable and needs to be approximated (ideal filters for example).

<u>Stability</u>

A LTI system is said to be stable if every bounded input gives rise to bounded outputs. Bounding may occur in amplitude or time. A bounded short impulse signal generating an amplitude-bounded continuous oscillation is not bounded.

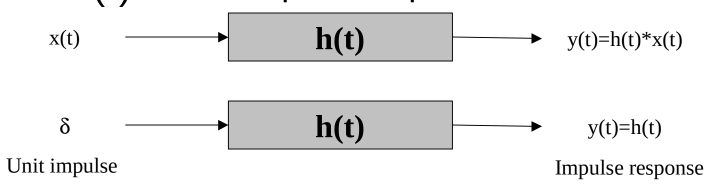
Filters, impulse response and convolution

A <u>continuous LTI</u> is called a linear <u>filter</u>

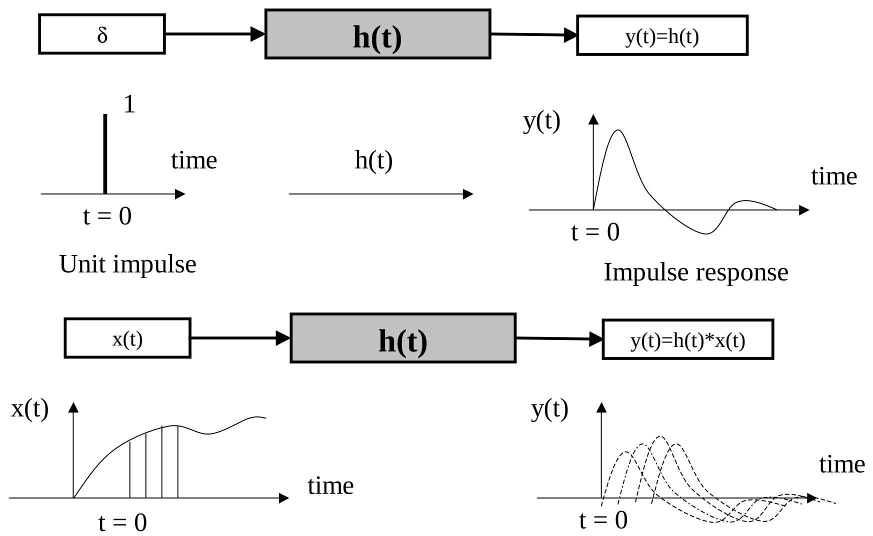
For an LTI the relation between input and output is *convolution*

$$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)x(\tau)d\tau = h(t) * x(t)$$

where $h(t)$ is the impulse response of the filter.



Graphical interpretation



Convolution theorem

Fourier Transform:

$$X(f) = F[x(t)] = \int_{-\infty}^{+\infty} x(t) e^{-2j\pi ft} dt \qquad x(t) = \int_{-\infty}^{+\infty} X(f) e^{+2j\pi ft} df$$

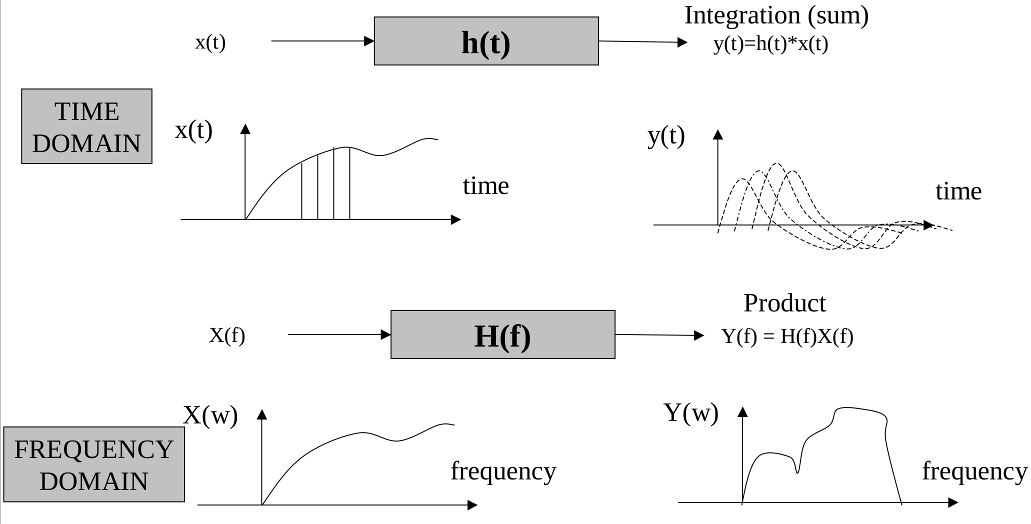
Convolution theorem:

$$F[x(t) * y(t)] = \int_{-\infty}^{+\infty} [h(t) * x(t)] e^{-2j\pi ft} dt$$

$$= F[x(t)] F[y(t)]$$

$$= X(f)Y(f)$$

Convolution theorem and LTIs



Convolution theorem and LTIs

Fourier Transform is KEY to signal processing and LTIs

Convolution theorem and LTIs

Graphical convolution $y(t) = \int_{-\infty}^{+\infty} h(t-\tau)x(\tau)d\tau = h(t) * x(t)$

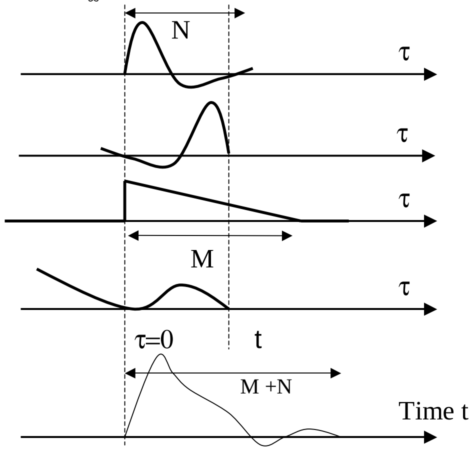
Impulse response h(t)

Time reversed and translated
Impulse response h(t)

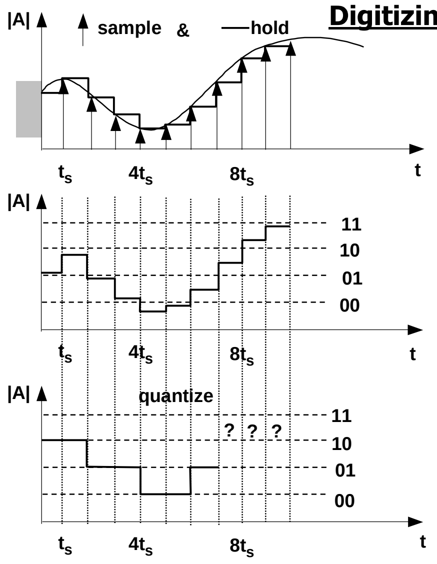
Input function x(t)

Product

Linear Convolution



Back to digital signal processing

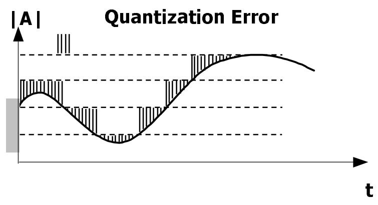


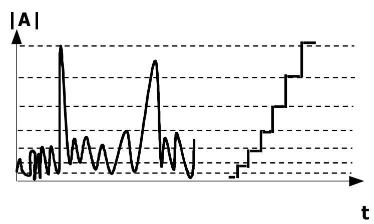
Digitizing the Signal

- Aim is to obtain 1, 0 representation
- Sample signal periodically
- Hold sampled value until next sample
- Classify new signal into levels (quantize the signal)
- More levels means more accuracy

Signal = 10 10 01 **01** 00 00 01

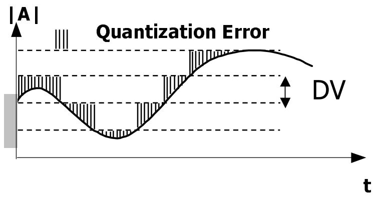
Quantization Error

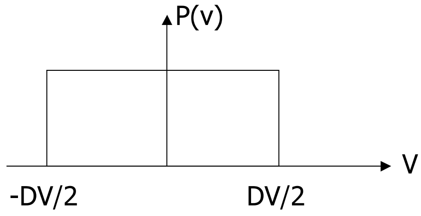




- Quantization introduces errors
- Increasing the number of quantization levels is not always the answer
- Non-uniform Quantization
  - Use more levels where there are more variations
  - Use fewer levels where there are fewer variations

Quantization Error (uniform quantization)





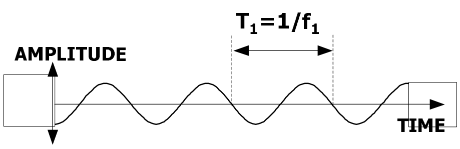
Probability density function
of a point in any interval

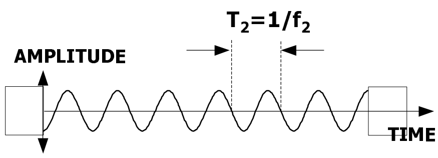
$$\text{Mean Square Error} = \frac{1}{\Delta V}\int_{\frac{-\Delta V}{2}}^{\frac{-\Delta V}{2}} v^2 dV = \frac{(\Delta V)^2}{12}$$

$$\text{RMS} = \frac{(\Delta V)}{2\sqrt{3}}$$

Sampling

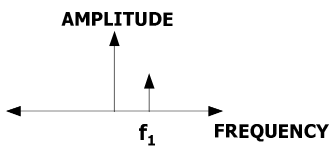
Time Domain

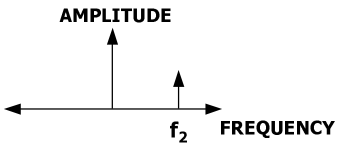




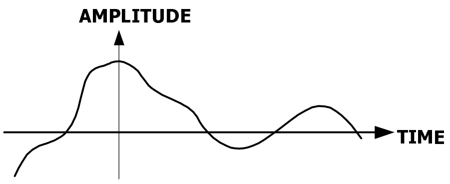
T = period **f = frequency**

Frequency Domain

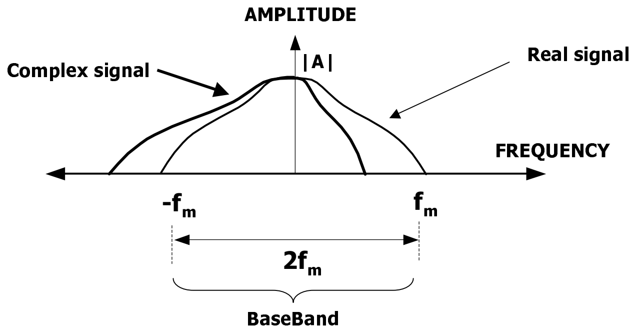




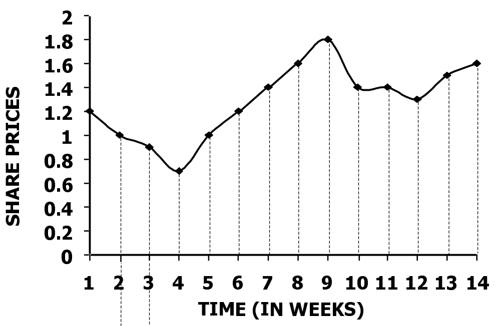
Real Signals



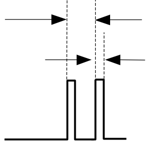
Real life signals are a combination of many frequencies



Sampling



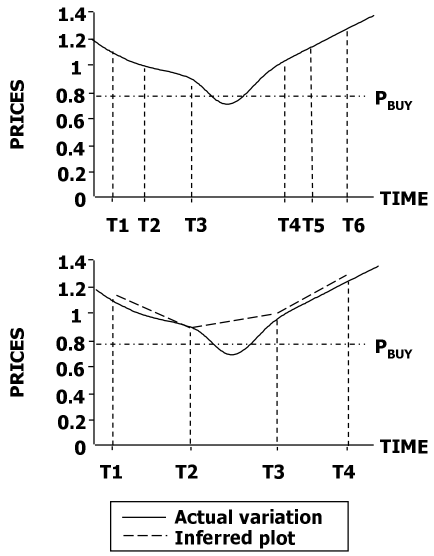
- **Take snapshots of continuously changing data**
- **The Sampling Period is fixed**
- **This makes information understandable**
- **My share price hit its lowest point in week 4**
- **My share price reached its peak in week 9**



Sampling Period – The period between samples

Sampling Time (The Snapshot) – The time taken to take a sample

Missing Information



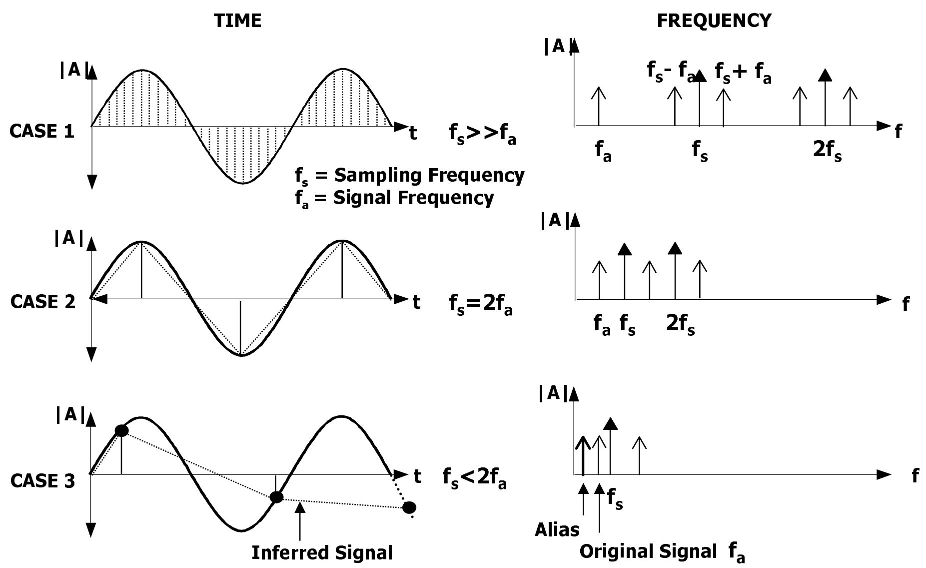
- **Non-periodic Snapshots**
  - **May miss information**

    The dip in prices between T3 and T4 goes unnoticed
  - **Information cannot be interpreted easily**
- **Periodic Snapshots**
  - **May miss information**

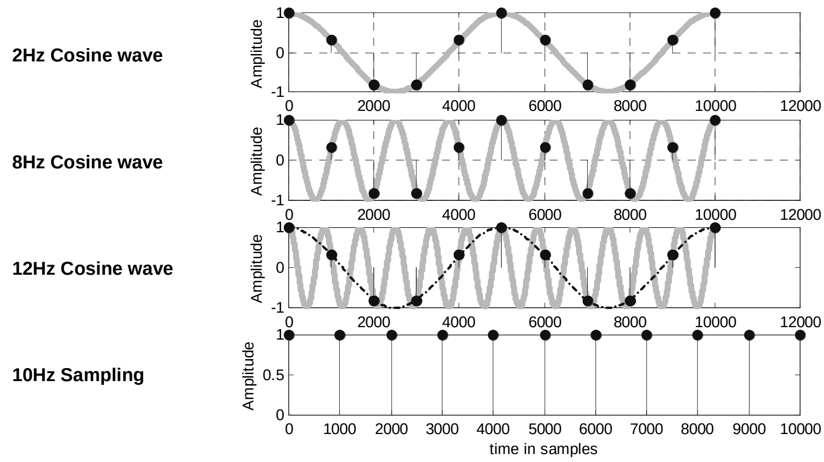
    The dip in prices between T2 and T3 goes unnoticed
  - **Easier to interpret**

<u>The key is the sampling frequency.</u>

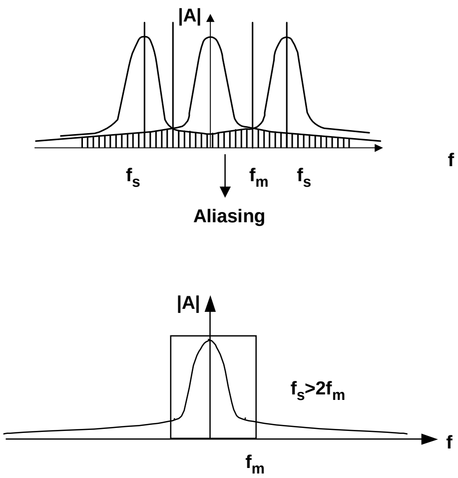
Getting the Sampling Right



Getting the Sampling Right



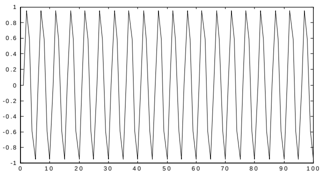
Limiting the Spectrum



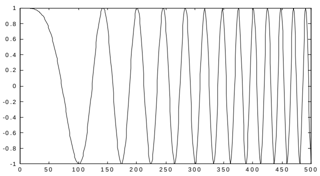
- Signals in the real world contain many frequencies
- Frequency components greater than $1/2f_s$ cause aliasing ($f>f_m$)
- Get rid of (filter out) frequencies above f_m (no aliasing)
- Then ensure that the sampling rate is greater than $2f_m$

Example of aliasing

Signal type	Signal characteristics	Sound produced
Sine wave	Fs = 1kHz	
	F = 200Hz	sine_200_1000.wav
	F = 400Hz	sine_400_1000.wav
	F = 600Hz	sine_200_1000.wav
	F = 800Hz	sine_400_1000.wav
Linear FM chirp	F = 0 to 1 KHz in 10 sec	chirp_1000_1000.wav



Sine wave



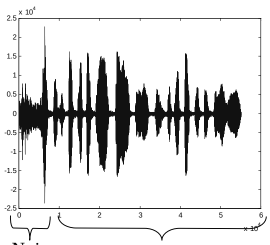
Linear FM chirp

Example of aliasing

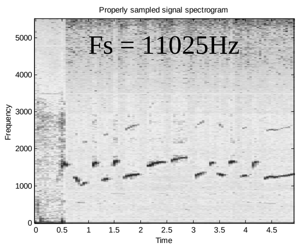
Signal type

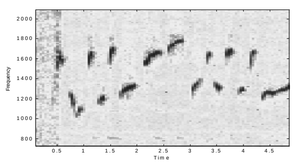
Spectrogram

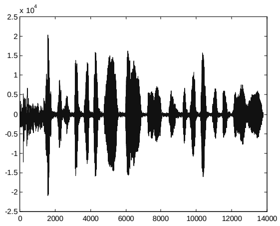
Sound produced

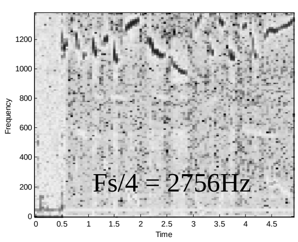


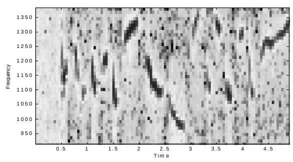
Noise whistle





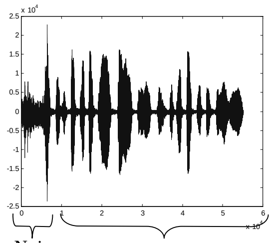


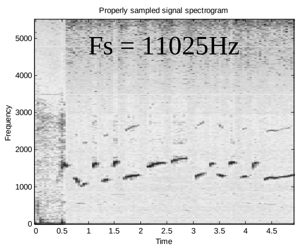


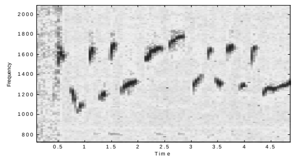


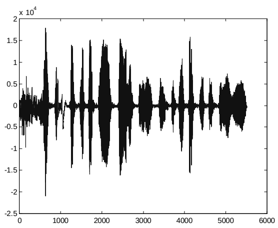
Example of aliasing

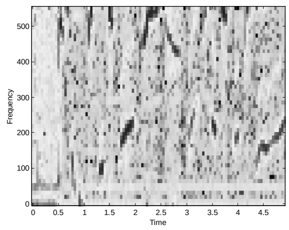
Signal type | Spectrogram | Sound produced











Fs/10 = 110.2Hz

Sampling revisited (with theory and math)

Math background:

Delta function:

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{elsewhere} \end{cases} \qquad \delta(x - x_0) = \begin{cases} 1 & \text{if } x = x_0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_{x_1}^{x_2} f(x)\delta(x - x_0)dx = f(x_0) \quad \text{with } x_1 < x_0 < x_2$$

$$f(x) * \delta(x) = \int_{-\infty}^{+\infty} f(\tau)\delta(x - \tau)d\tau = f(x)$$

$$f(x) * \delta(x - x_0) = f(x - x_0)$$

$$F[\delta(x)] = 1$$

$$F[\delta(x - x_0)] = e^{-2j\pi f x_0}$$

Sampling revisited (with theory and math)

Application examples

Fourier transform of a sine?

Fourier transform of a cosine?

Inverse Fourier transform of a pure frequency?

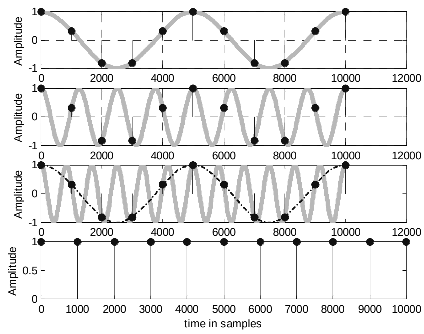
Negative frequency concept?

Sampling revisited (with theory and math)

Comb function:

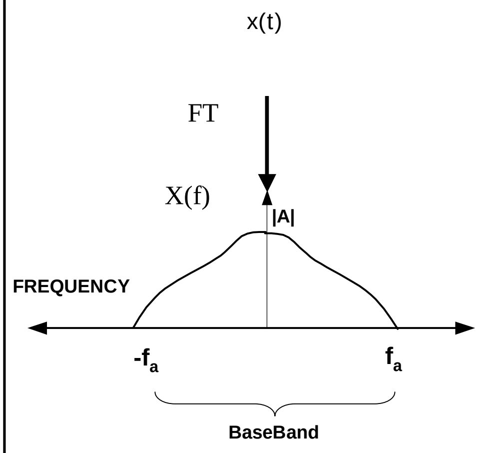
$$\mathrm{comb}(x) = \sum_{k=-\infty}^{\infty} \delta(x-k)$$

$$\mathrm{comb}\left(\frac{x-x_0}{b}\right) = |b| \sum_{k=-\infty}^{\infty} \delta(x - x_0 - nb)$$



$$F[\mathrm{comb}(t/T)] = F\left[\sum_{k=-\infty}^{\infty} \delta(t-kT)\right] = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

Sampling revisited (with theory and math)



$$x_s(k) = x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT) = x(t)\mathrm{comb}(t/T)$$

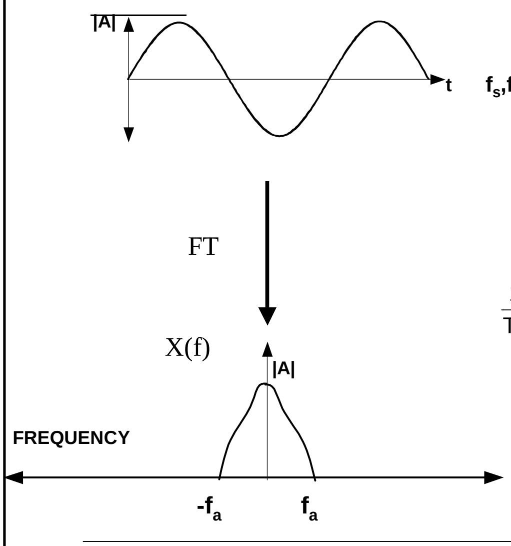
FT

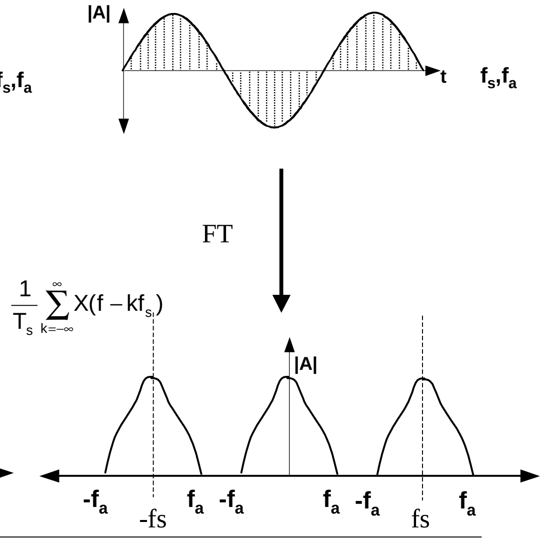
$$X_s(f) = F\left[x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s)\right] = F[x(t)] * F[\mathrm{comb}(t/T_s)]$$

$$= X(f) * \frac{1}{T_s}\sum_{k=-\infty}^{\infty}\delta(t-\frac{k}{T_s}) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty}X(f) * \delta(t-kf_s)$$

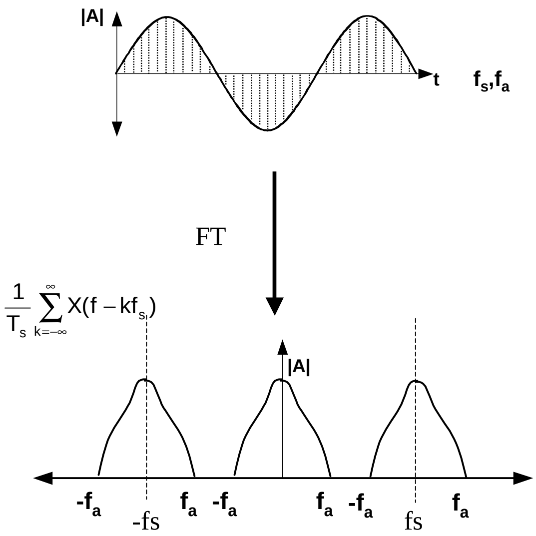
$$= \frac{1}{T_s}\sum_{k=-\infty}^{\infty}X(f-kf_s)$$

Sampling revisited (with theory and math)





Sampling revisited (with theory and math)



$f_s < 2f_a$?

$f_s = 2f_a$?

$f_s > 2f_a$?

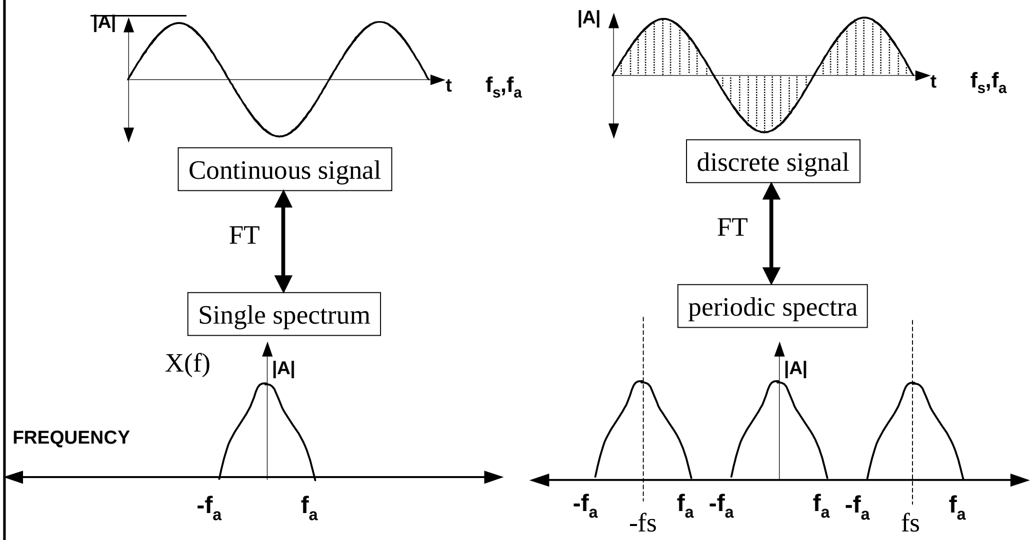
Nyquist's sampling theorem

If the highest frequency component in a continuous signal is f_{max}, then that signal can only be recovered from its data samples if the sampling frequency f_s is greater than twice f_{max}.

<u>Examples:</u>

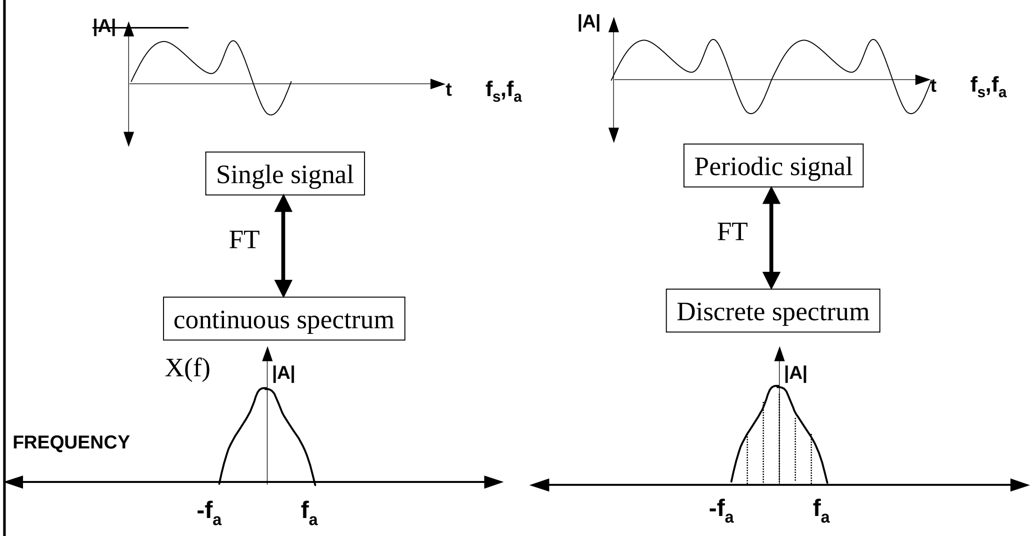
Audio CD: [0-20kHz] Sampling at 44 kHz (why not 40?)

Discreteness and periodicity



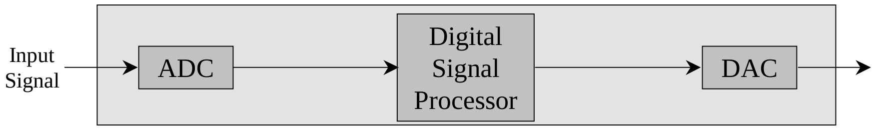
Sampling and discretization 1.35

Discreteness and periodicity



Sampling and discretization 1.36

ADC revisited



Move signal to baseband?

choose sampling frequency f_s

- Based on information required in signal
- Based on existing ADC converters and design tradeoffs (cost, power dissipation)

Low pass filter signal to half (Anti aliasing filter)

Sample signal at f_s

Learning outcomes

Definition and properties of Linear Time Invariant (LTI) systems

Filtering and the convolution theorem

Fourier transform and convolution

Digitization

Sampling and aliasing. Nyquist theorem

Sampling and periodicity: **Sampling in one domain is periodizing in the other**

ADC conversion: anti-aliasing filtering followed by sampling

Worked Examples: Digital Radio System

A digital radio system consists of a transmitter on one building and a receiver on another building. The impulse response of the receiver and the transmitter are the same given by figure 1 below. A logic 1 (High) is transmitted applying a unit impulse to the transmitter. A logic 0 (low) is transmitted by sending a negative impulse (-1) to the transmitter. The time taken fro the transmitted pulse to reach the receiver is negligible.

(a) Draw a block diagram of the system. Using convolution, make a detailed sketch of the the waveform at the output of the receiver when a logic 1 (High) is transmitted.

(b) A Logic 0 is transmitted 0.5 ms after a logic 1. In other words the signal:

$$\delta(t) - \delta(t - 0.5x10^{-3})$$

is applied to the transmitter. Make a detailed sketch of the waveform at the output of the receiver.

(a) In bad weather, there are two path between the transmitter and the receiver. The first path (direct) has a negligible time delay and a gain of 1. The second path has a gain of −1 (reflection) and a delay of 0.25ms. The contributions of the two paths add together at the input of the receiver. Draw a diagram of the complete system. Make a sketch of the waveform output of the receiver when a logic 1 is transmitted.