Unsupervised registration of textured images: applications to side-scan sonar

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***Abstract*— Sonar images are highly textured images and therefore mislead most of the classical registration algorithms. Registration is a critical step for the creation of high-resolution accurate mosaic images of the seafloor required for seabed analysis and classification. In the past concurrent mapping and localisation have successfully been used but the detection and association of landmarks have been proved difficult and been done manually. However, such methods are time consuming and lack robustness. Landmarks are not regularly present in the images and their localisation is prone to errors. As a consequence, global methods using whole images are preferable. These methods were extensively studied in the recent years and successfully applied to multi-modal medical image registration. Unfortunately, the similarity metric between images they rely upon cannot cope with highly textured images. To overcome this issue, textural features must be extracted to highlight similar regions of the images. Registration of these feature maps works but remains sensible to the feature selection and their relation from one modality to the other. An alternative approach is proposed in this paper. Mutual information is calculated from all the features and global registration can be achieved directly. Solely an approximation of MI can be obtained but the performance of this algorithm are equivalent to exact approach and robust to feature selection. This method has been successfully applied to textured images (side-scan sonar) but is also applicable to multi-modal images such as bathymetric and sonar data.**

I. REGISTRATION IN A MARINE CONTEXT

Offshore technologies and geo-sciences present increasing needs for an accurate description of the seabed. The applications are as various as human impact assessment, novelty detection and mapping and lead to the generation of huge amount of data which must be automatically post-processed. These underwater studies use the deployment of several sensors (e.g. bathymetric, side-scan sonar) to access physical and geometrical measurements about the seafloor. In addition, for novelty detection, these acquisitions are performed periodically.

Given several images (different viewpoints, modalities, acquisition dates) of the same seafloor, a mandatory preliminary step consist of registering these images before any further processing (e.g. classification, objects detection). Registration ensures that pixels of equal coordinates on different images correspond to the same point of the scene.

Mosaics of seafloor using navigation information generally presents large errors due to the inaccuracies of underwater localisation. Concurrent mapping localisation [1] have been successfully used but detection and association of landmarks have been achieved manually. Automated registration approaches can be split into two families. On one hand, point matching techniques [2] model the scene, through the matching of landmarks. These landmarks are often points, detected in expected meaningful areas [3]. Automated landmarks extraction in underwater images, and in side-scan sonar images especially, is made difficult by strong textures and noise (e.g. interference, first return) preventing us to use landmark-based methods.

On the other hand, global methods [4] estimate the best registration by using all the pixels of the images. The most successful among these methods remain the measure of a similarity metric (pixel-based distance between overlapping regions of the image). The latest is maximised regarding a geometrical transformation [5], [6], which is chosen according the expected distortion of the images.

Under the assumption of having a flat seabed, global methods are suitable to perform the registration. They match regions within the images leading to a better estimate of the registration. Regions are reliable features of the seabed because they are observable from any modalities and are constant over time. Three similarity metrics [5], [6], [7] were proposed in the literature. They rely on different models of change of the grey levels from one image to the other. However, highly textured images mislead these algorithms. These methods must be assessed in a marine environment (textured and multi-modal images).

This paper presents how textured images can be registered using Mutual Information (MI) and texture features. These preliminary results are promising to extend this method to multi-modal images. Section II recalls the principle of these techniques. Textural features are then extracted from the images as shown in section III. In order to perform registration using MI on multichannel images (the feature images) an estimate of MI in this new high-dimensional space must be derived. This is done in section IV. This estimator of high-dimensional MI is evaluated in section V where results on real images are presented. Finally, conclusions on the method and future work are given in section VI.

II. MUTUAL-INFORMATION BASED INFORMATION

A. Similarity maximisation based registration

Among pixel-based registration methods, FFT-based [8] and similarity-based [5] are the most extensively used. The second one was successfully applied to multi-modal medical images registration [9], [5], [10], [6], [7]. This approach has the great advantage to be able to use any deformation models, from rigid-body transform to warping model [11]. Given a similarity metric μ between images, the registration procedure aims at increasing the similarity between a reference image I and $T(J)$, the image to register transformed with T, by adapting T (1).

$$\hat{T} = \underset{T \in \mathcal{T}}{argmax}\ \mu(I, T(J)) \tag{1}$$

The transform T that maximises this similarity is sought within a family of geometrical transform $\mathcal{T}$. This set is chosen regarding the possible alterations existing between two images (e.g. rigid-body transform, warping). Typically, these transform are parametrised with a parameter vector $\overrightarrow{\alpha}$ leading to the following optimisation problem:

$$\hat{\overrightarrow{\alpha}} = \underset{\overrightarrow{\alpha}}{argmax}\ \mu(I, T_{\overrightarrow{\alpha}}(J)) \tag{2}$$

B. Similarity metric between images

The similarity metric is selected according to the image formation models, i.e. how the grey level of the same point of the scene is modified from one image to the other. The sum of square difference SSD [11] assumes that luminance is preserved. Correlation Ratio CR (3) [6] simply require a functional dependence between the grey-level. This simple model allows to perform registration on multi-modal medical images (MRI, CT, PET [6]).

$$CR(I, T(J)) = \frac{var(E[J|I])}{var(J)} \tag{3}$$

To cope with more complex image formation model, Mutual Information [5] is a information-theoretic measure that only expects a "predictable" relation between the two images. The measure of the correlation between the images with MI rely on the joint histogram allowing more complex relations between grey levels. Typically, a grey level of one image can be represented by several grey levels in the other.
MI is based on Shannon entropy H and is given in (4). The maximisation of MI mainly corresponds to the minimisation of $H(I, T(J))$, i.e. the increase of the correlation between I and $T(J)$ in a statistical sense.

$$MI(I, T(J)) = H(I) + H(T(J)) - H(I, T(J)) \tag{4}$$

Similar metrics were developed afterwards to improve the registration results. Instability due to the size of the overlapping area were reduced with normalised version of MI [12], [10]. A new information measure, called Cumulative Residual Entropy [13], based on the cumulative distribution function instead of the probability density function, can replace the entropy defined by Shannon. The use of this measure in the definition of MI leads to the definition of the Symmetric Cross Cumulative Residual Entropy SCCRE [7] which is more robust to noise.
All these metric are suitable to perform registration but are solely adapted to ideally piece-wise constant images. Textured images are not in generally properly registered simply based on the grey levels. Indeed, texture leads to several local maxima of MI which prevents a numerical maximisation scheme to find the global maximum. MI remains the most general metric and is preferred for its ability to cope with complex relationship between images.

III. Textural features

A. Grey Level Co-occurrence matrices

Grey levels of textured images are not relevant for registration because they fail to describe regions. The classical method to reveal information about regions (e.g. for segmentation) consists of extracting textural features, chosen to be constant over similar regions. In the case of multi-modal images, other features can be chosen depending on the type of image. For instance, for MRI and any other piecewise constant images, grey levels are relevant features. In the case of textured images, various features [14], [15] were proposed in the literature.
Grey level co-occurrence matrices $\{p(i,j)\}_{0 \leq i,j \leq L-1}$ are local statistical descriptions of neighbourhoods in images with L grey levels. This second order statistics of the grey-levels is a classical descriptor of texture, which measures the joint probability density of the grey levels of one pixel and one of its neighbours. The latest is chosen in one direction at a given distance δ. In our case, this matrix is computed for each pixel along four directions ($\phi = -90°, -45°, 0°, +45°$). δ depends on the texture present in the image.
Instead of working with L^2 values for each matrix, Haralick [14] proposed a set of 14 features, extracted from the matrix, describing the texture. We use particularly two features among the Haralick feature set given thereafter. Therefore, we have access to 8 features to perform the registration.

1) Energy

$$f_1 = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p(i,j)^2 \tag{5}$$

2) Contrast

$$f_2 = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} (i-j)^2 \cdot p(i,j) \tag{6}$$

Figure 1 presents textural features. Figure 1(a) is the grey level images and (b-c) are the images of the two previously given features for the co-occurrence matrix for $(\phi = -90°, \delta = 2)$.

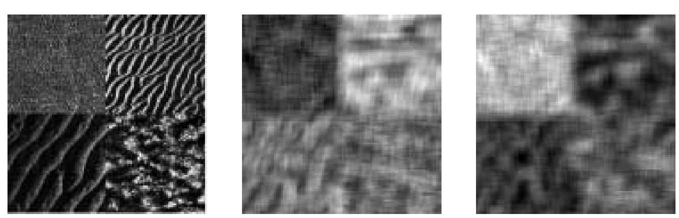


Fig. 1. Textural features: (a) Grey-levels (b) Energy (c) Contrast

In figure 1(b-c), identical textures appears with similar feature values. These piece-wise constant images can be registered using the similarity metric presented in section II. Although, registration relying on only one feature might fail because of the impossibility to discriminate any texture with only one feature. For instance, energy provides similar measures for the two lower textures. Therefore, a seducing approach to spare ourselves the issue of feature selection would use simultaneously all the features to perform the registration. The robustness of the registration is then increased because the feature extraction remains less critical.

B. Concurrent registration of the feature maps

Each images is now represented with several channels, the extracted feature maps. Equations (1) and (4) are still valid. Each pixel is modelled as a independent multivariate random variable $\overrightarrow{F}$ (feature vector). Given P the probability density function of $\overrightarrow{F}$, Shannon entropy is given by:

$$H(\overrightarrow{I}) = -\sum_{\overrightarrow{F}\in\mathcal{F}} P(\overrightarrow{F}) \cdot \log_2(P(\overrightarrow{F})) \qquad (7)$$

Unfortunately, the estimation of P over the feature space $\mathcal{F}$ is not possible for two reasons. First, working on a N^d histogram, where N is the number of grey level and d the number of features, is computationally intractable. Second, and crucially, the scatter plot of the image in $\mathcal{F}$ is too sparse to allow an accurate estimation of P with classical methods, i.e. with binned or kernel based [16] estimator.

This limitation is known and three approaches were proposed in the literature to register multi-channel images. A principal component analysis PCA can be done [17] to combine the features into a single grey-level images which is going to be registered. Textured images cannot be handle with such methods though. Using the extracted features, images can be segmented and then registered at the class map level using MI [18]. But, registration of side-scan sonar, and more generally textured images, is still an open and widely investigated problem. The third methods was recently proposed by Kybic [19] and consists of approximately estimating the entropy in this high-dimensional space.

This more straightforward method use all the information available. In addition, any features can be used (e.g. texture, neighbouring pixels grey values), independently on both images. This really promising approach for underwater images registration (i.e. textured and multi-modal) is presented in the following section.

IV. High-dimensional Mutual Information

A. Approximation of the entropy of continuous random variables

Equation (7) can be approximated by:

$$\begin{aligned} H &= -E[\log_2(P(\overrightarrow{F})] && (8) \\ &= -\sum_{\overrightarrow{F_i}} \log_2(P(\overrightarrow{F_i})) && (9) \end{aligned}$$

where $\{\overrightarrow{F_i}\}_{i\in[1,N]}$ are N realisations of $\overrightarrow{F}$.

When $\overrightarrow{F}$ is a continuous random variable, Kozachenko and Leonenko [20] proposed an estimator of H using the correlation between $P(\overrightarrow{F_i})$ and the local density in the feature space $\mathcal{F}$. The local density can be measured by the Euclidean distance to the nearest neighbour of $\overrightarrow{F_i}$ in $\mathcal{F}$, noted λ_i. This intuitive description leads to the equality (10). Given (9), the entropy can be approximated by (11).

$$\begin{aligned} -\log_2 P(\overrightarrow{F_i}) &= dE[\log_2(\lambda_i)] + \\ & \underbrace{\log_2\left[\frac{(N-1)\pi^{d/2}}{\Gamma(\frac{d}{2}+1)}\right] + \frac{\gamma}{\log_e(2)}}_{\kappa(N,r)} && (10) \\ H^*(\overrightarrow{I}) &= \frac{d}{N}\sum_{i=1}^{N} E[\log_2(\lambda_i)] + \kappa(N,d) && (11) \end{aligned}$$

To estimate $P(\overrightarrow{F_i})$ by measuring an Euclidean distance is really convenient. The distance to the nearest neighbour is unique and several realisations of it are not available usually. Therefore, Kozachenko and Leonenko proposed to estimate the entropy with:

$$H_{KL} = \frac{d}{N}\sum_{i=1}^{N} \log_2(\lambda_i) + \kappa(N,d) \qquad (12)$$

where pixels with $\lambda_i = 0$ are discarded of the sum.

This estimator is asymptotically unbiased, i.e. $E[H_{KL}(X)] - H(X) \sim O(1/\sqrt{N})$, where N is the total number of samples. The penalty paid for this lack of bias is a reduced precision when compared to binned estimates. Nevertheless, acceptable values of entropy are obtained with a few samples ($\simeq 100$, [21]).

This estimator has been successfully applied to physiological studies [21], and was recently proposed for registration purpose [19]. However, its adaptation to the discrete case is not straightforward. Discretisation of the feature space leads to the discretisation of λ_i, which loses its accuracy and reliability as an estimate of the local density. $\lambda_i = 0$ is equivalent to an infinite probability density. As a consequence, the estimator becomes biased due to the large amount of samples not taken into account for the estimation.

B. Discrete estimator of Kozachenko and Leonenko

Kybic [19] proposed to take advantage of the additivity of entropy to estimate Mutual Information on the N samples. Theses samples are randomly subsampled into $[N/M]$ subsets, or batches, of M samples. On each, MI is estimated with the estimator (12), with a brute-force nearest neighbour search to estimate λ_i.

His method benefits of a overall decrease of the local density in $\mathcal{F}$ for the estimation of MI. The occurrence of $\lambda_i = 0$ becomes rare and the estimator of H reliable. This stochastic estimation depends on the parameter M though. A trade-off must be chosen between accuracy, enough samples in the batches, and sufficient subsampling. A drop of performance of this approach is observed when M increases, instead of a gain of accuracy, because of the biggest part of non-informative samples ($\lambda_i = 0$) in the batches. $M = 100$ appears to be an appropriate parametrisation in most of the case.

C. Registration algorithm

Our implementation of the registration assumes a rigid-body transform between two images of the seafloor. The family $\mathcal{T}$ (2) of parametrised transform can be written under the form:

$$T_{\overrightarrow{\alpha}}(\overrightarrow{y}) = \frac{1}{s} \cdot R(\theta) \cdot \overrightarrow{y} + \overrightarrow{t} \tag{13}$$

where the parameters $\overrightarrow{\alpha}$ are rotation θ, translation $\overrightarrow{t}$ and scale s.
The estimation of $T_{\overrightarrow{\alpha}}(\overrightarrow{J})$ is done with a bi-linear interpolation. The maximisation of the similarity measure is performed with a simplex algorithm. This multi-dimensional and unconstrained algorithm uses the measure of MI for $\gamma + 1$ values of the parameter $\overrightarrow{\alpha}$ (γ is its dimension) and locally approximates the similarity measure by the facet these $\gamma + 1$ vertices generate. The registration algorithm can be summarised as:

<u>*Registration algorithm*</u>

1. Feature Extraction
2. Estimate $\mu(\overrightarrow{I}, T_{\overrightarrow{\alpha}}(\overrightarrow{J}))$ for an original set of vertices
3. Given the facet, update the set of vertices to maximise MI
4. Estimate $MI(\overrightarrow{I}, T_{\overrightarrow{\alpha}}(\overrightarrow{J}))$ for these $\gamma + 1$ values of $\overrightarrow{\alpha}$
5. If the facet collapses into a point of the parameter space. Then, end. Else, go to [3]

The assessment of image processing algorithm is generally a difficult problem, and registration is not an exception. Usually, the unique quantitative results associated are usually the measure of the error made on the estimation $\overrightarrow{\alpha}$. However, this error is poorly associated to the geometrical distortion of $T_{\overrightarrow{\alpha}}(\overrightarrow{J})$. We propose to estimate the geometrical alteration of the image due to the error made on the parameters to evaluate our approach. This registration quality factor ε corresponds to the worse error made on the predicted position of one pixel. Mathematically, ε is the upper-bound of $\|\partial T / \partial \overrightarrow{\alpha}\|$. The simple transform (13) taken into account allows us to calculate ε:

$$\begin{aligned} \frac{\partial T_{\overrightarrow{\alpha}}(\overrightarrow{y})}{\partial \overrightarrow{\alpha}} &= \frac{1}{s} \cdot R(\theta) \cdot \left[\frac{-1}{s} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \overrightarrow{y} \cdot ds \right. \\ &\left. + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot (\overrightarrow{y}) \cdot d\theta \right] + d\overrightarrow{k} \quad (14) \\ \left\| \frac{\partial T_{\overrightarrow{\alpha}}(\overrightarrow{y})}{\partial \overrightarrow{\alpha}} \right\| &\leq \left\| \frac{R(\theta)}{s} \right\| \cdot \left[\frac{1}{s} \|\overrightarrow{y}\| \cdot |ds| + \right. \\ &\left. \|\overrightarrow{y}\| \cdot |d\theta| \right] + \|d\overrightarrow{k}\| \quad (15) \end{aligned}$$

Let l be the largest dimension of the image. Let note $|dk| = \max_i |dk_i|$. Given $\|R(\theta)\| = 1$, (15) becomes:

$$\left\| \frac{\partial T_{\overrightarrow{\alpha}}(\overrightarrow{y})}{\partial \overrightarrow{\alpha}} \right\| \leq \frac{l}{s} \cdot \left(\frac{|ds|}{s} + |d\theta| \right) + |dk| \tag{16}$$

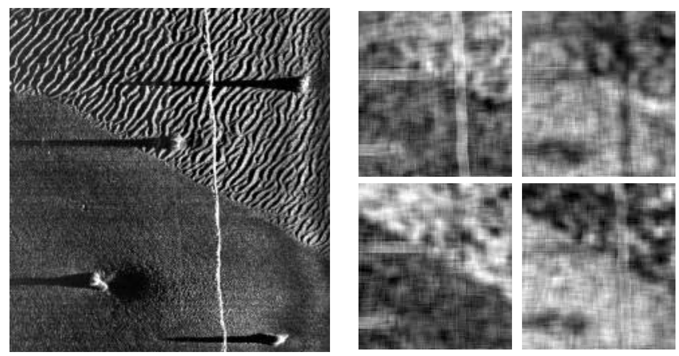
$$\epsilon = \frac{l}{s} \cdot \left(\frac{|ds|}{s} + |d\theta| \right) + |dk| \tag{17}$$

(17) is the relative error used throughout this paper to compare registration techniques.

V. Results

A. Side-scan sonar images

Figure 2(a) represents a template of side-scan sonar images. The right hand side of the figure shows 4 out of the 8 feature maps we extract.



(a) (b)

Fig. 2. Side-scan sonar image template (a) Image (c) 4 feature maps (energy -left- and contrast -right- for co-occurrence matrices measured at 0° -top- and 45° -bottom-)

In a preliminary experiment, this template is artificially transformed according to the transform (13), to get ground-truth images to register. The various transform applied have solely one parameter modified, the others remaining at 0 (or 1 for the scale). This leads to 19 synthetic registration problems, with ground-truth, where the performance can be assessed through the measure of ε (17).
Features are then extracted ($\delta = 2$) and the registration is performed following the algorithm given in section IV-C. To register the images, exact calculation of MI is performed using the energy of the co-occurrence matrix measured at $-90°$. This feature seems to efficiently describe the image into regions. High-dimensional MI is estimated on the 8 feature maps extracted.
Table I summarises the results obtain with different similarity metrics. Exact calculation of similarities given in section II and the two high-dimensional MI estimators are compared during registration task.
First of all, classical registration based on grey level images fail in this case even with high contrast. Periodic texture, common in side-scan sonar image, make the similarity bumpy. These local maxima mislead the optimisation algorithm. This table shows that co-occurrence matrices is an efficient way to discriminate these textures. Classical exact calculation of CR

TABLE I

REGISTRATION ERROR ε FOR 5 SIMILARITY METRICS

Metric	CR	MI	SCCRE	HD-MI
Angle (in degree)				
5	1.7	*673*	1.5	0.8
10	**0.6**	*573*	0.7	**1.5**
15	*49*	*554*	**1.5**	*18*
20	*127*	*558*	*287*	*143*
Translation (in pixels)				
5	1.1	**2.4**	1.0	1.0
10	**0.4**	*433*	**0.5**	**0.8**
15	*48*	*451*	*73*	*44*
Scale (no unit)				
1.2	**2.7**	*385*	**1.8**	*43*
1.4	*67*	*238*	*66*	*77*

and SCCRE leads to accurate registration of this image with manually selected feature. Mutual Information fails to register though.

High-dimensional MI provides equivalent results than these approaches. The approximation of mutual information on all the feature maps provides a trustworthy alternative to exact metric calculation. The highest computation cost is balanced by the saving of a feature selection stage.

B. Multi-view side-scan sonar

The previous example is not representative of a real registration case. Figure 3 presents two views of the same area (the images are inverted for the sake of clarity). These images are convenient because they are acquired from the same viewpoint with different sonars.

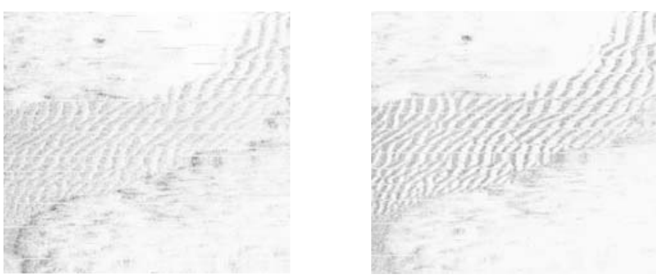


Fig. 3. Two views of identical area (inverted images)

The contrast these images is much worse than the previous case and the feature maps measured for $\delta = 10$ (figure 4) are much less ideal than in the previous case.

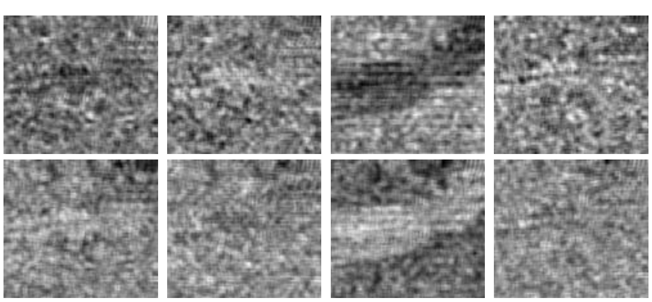


Fig. 4. Feature map of figure 3 (left). Top line=energy. Bottom line=contrast. From left to right: $\phi = -90°$, $-45°$, $0°$, $+45°$

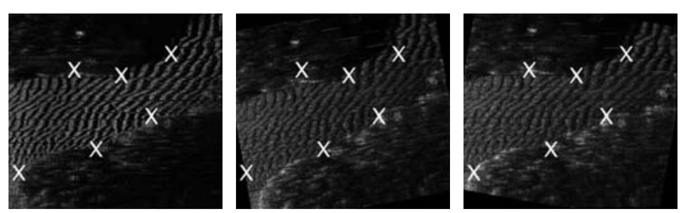
The same experiments are performed than in the previous case. Table II summarises the registration performances of these methods. The registration is performed 5 times, twice with CR and SCCRE and once with high-dimensional MI. The first registration, case #1, using CR and SCCRE is performed using the energy of co-occurrence matrix at $\phi = -90°$, as selected previously. Then, case # 2, the energy maps measured on two different co-occurrence matrices ($\phi = -90°$, and $\phi = 0°$) is used to perform the registration of the images of figure 3.

TABLE II

REGISTRATION ERROR ε TO REGISTER FIGURE 3

	Case #1		Case #2		HD-MI
Metric	CR	SCCRE	CR	SCCRE	Kybic
Angle (in degree)					
5	10.8	*35*	*43*	*18*	12.7
10	10.7	10.9	*102*	*100*	**14.1**
15	**12.7**	**12.4**	*148*	*116*	*42*
20	*139*	*204*	*190*	*183*	*341*
Translation (in pixels)					
5	10.0	11.5	*53*	*54*	10.6
10	11.5	12.3	*68*	*70*	10.3
15	11.1	11.2	*80*	*79*	11.6
20	10.2	**9.4**	*109*	*89*	11.1
25	**10.6**	*60*	*54*	*82*	**12.4**
30	*69*	*112*	*88*	*90*	*50*
Scale (no unit)					
1.2	**11.1**	**10.9**	*83*	*92*	*22*
1.4	*76*	*95*	*144*	*148*	*99*

Registration error has increased ($\varepsilon \simeq 10$), in comparison of the previous case. The features are less discriminant due to the lowest contrast of the image. Figure 5 presents qualitatively the results of the registration of the images, rotated of $10°$. The reference image on the left and the corrected image, with high-dimensional MI, appears similar. The 6 white markers represent 6 pixels with the same coordinates. They are not used for registration but shown to help the reader assess the quality of the registration. Despite the increase of the quantitative error, the results are qualitatively good.



(a) (b) (c)

Fig. 5. (a) Reference Image (b) Image to register (c) Corrected Image

Table II shows that CR and SCCRE still perform well in the first case. This approach is slightly better than the estimation of high-dimensional MI. The parameter δ was adapted to the type of texture but the feature selection is robust to change of illumination (same viewpoint, same modality). However, as soon as the features change (case #2), both metrics fail to uncover similarities of the images.

In a known environment, side-scan to side-scan registration can be performed using a feature extraction and classical MI-like methods [5], [6], [7]. However, robustness to feature selection is sought to tackle other registration issues (e.g. multimodal, multiresolution). We proposed to use an estimator of the mutual information of all the feature maps [19]. This approach provides a simple mechanism to take into account all the extracted features. Even if this technique relies on an approximation, similar results than in the exact metric measure were obtained.

VI. Conclusions and future work

Registration of underwater images are a very important topic with the growth of high resolution marine science mapping requirements. Off-shore industries and geo-sciences need a better description of the seafloor and perform extensive surveys of entire areas. The huge amount of produced data requires automated processing to generate exhaustive maps of the seabed. This paper focused on the registration of underwater images using global registration methods and presented initial results.

Global methods, based on similarity measures, are promising methods in the underwater context. These region-based registration scheme rely upon areas which are observable from different view-points and modalities and constant over time. These methods are applicable in seabed survey as soon as the assumption of flat seabed is correct.

The classical similarity metrics applied to these grey-level images fail to recover geometrical transformation because of texture information. Relevant information for registration must be extracted to highlight the region of identical types of seabed. In side-scan sonar, textural features appears naturally as characteristics of the type of seabed. In this paper, co-occurrence matrices and Haralick feature set [14] have been successfully used to discriminate sea-bottom textures, but require the prior knowledge of the typical period of periodic textures.

Side-scan sonar images registration were preliminarily studied. Two convenient methods were applied for registration of feature maps. Exact similarity calculation based on a single feature has been successful but requires a feature selection stage. Another method is based on the estimation of high-dimension estimate of MI and showed equivalent performances than exact calculation. Beyond the higher computational cost, this method does not require a feature selection. More complex registration issue (e.g. multimodal) can be tackled by simply using all the feature maps available.

Future work is two fold. First, better textural features would be sought to prevent the need of prior related to the texture. Among textural features, wavelet frames [15] seems promising by incorporating a scale parameter related to the texture. Second, multi-modal registration would be studied within this framework, especially in the case of bathymetry and side-scan sonar.

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