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Pattern Recognition Letters 16 (1995) 843-848

Pattern Recognition Letters

Image processing optimization by genetic algorithm with a new coding scheme

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Received 21 December 1994; revised 3 April 1995

Abstract

An original coding scheme is introduced to take advantage of the two-dimensional structural information of images within the genetic algorithm framework. Results are presented showing that this new technique outperforms classical optimization methods for the optimization of 32 × 32 and 128 × 128 holograms.

Keywords: Optimization; Genetic algorithm; Image processing

1. Introduction

In the last ten years, iterative optimization techniques have been applied in a wide range of domains, from electronic circuit design to image processing. These techniques are particularly adapted to nonlinear multi-dimensional problems where no analytical solutions can be found and where the search space is too large for combinatorial search.

This paper compares some of these techniques in the particular application of computer generated holograms which have arisen as a very promising domain in the field of reconfigurable optical devices with applications in telecommunication and network design. The main problem in this case is that a fully complex transfer function, which is the optimal solution cannot be displayed on the currently available fast ferroelectric liquid crystal (FLC) electro-optical devices for which the coding domain is limited to binary images only. Iterative techniques therefore have to be applied to find the optimal binary image whose Fourier transform (called *reconstruction* of the hologram) gives a determined image (called *target*).

A new optimization technique is proposed here which outperforms all the other ones. It augments the genetic algorithm with a new coding scheme that takes advantage of the particular two-dimensional topological structure. The other optimization techniques, indeed, from hill climbing to simulated annealing, do not take this particular spatial structure of the images into account.

Although this new method is presented here only in the context of computer generated holograms, it can easily be extended to many other applications of image processing, particularly to image reconstruction.

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SSDI 0167-8655(95)00044-5

2. Problem formulation

Let X be an image of dimension $N \times N$, N being a power of two, corresponding to a hologram encoded as a binary phase image, i.e. with values 1 or -1 only.

Let T denote another image which can be grey-leveled and complex, the *target*.

The problem is to find the optimal hologram X in D, the space of binary images, which minimizes the *mean square error* between its Fourier transform and the target T:

$$X = \underset{Y \in D}{\operatorname{argmin}}(\|(F(Y) - T)\|^2) \tag{1}$$

where F denotes the Fourier transform.

A natural solution to this problem would have been the inverse Fourier transform of T. But this solution has to be rejected since it cannot be implemented on the available opto-electronic devices. The search space D is indeed restricted to binary phase images compatible with available technology. Moreover, there is no analytical solution to this constrained problem and a combinatorial search cannot be handled successfully since the number of possibilities grows as $2^{N \times N}$ and each evaluation of a solution involves computing a fast Fourier transform. To find an "optimal" solution to such a problem, iterative optimization techniques are the only alternative.

Every iterative scheme requires the definition of a search space and a cost function. In our case, the search space is D, the space of binary images, and the cost function is C, the mean square error between the Fourier transform of the candidate X and the target image T:

$$C(X) = \|(F(X) - T)\|^2. \tag{2}$$

3. Direct binary search

Direct Binary Search is a basic application of hill climbing to this problem. It can be decomposed into the following steps:

1. Initialize the search with a random binary image X of size $N \times N$.
2. Compute $C(X)$ the cost function of X.
3. Pass through all the pixels of X, row by row, trying to modify their value and keeping this modification when it leads to a lower cost. This is the basic iteration of the algorithm.
4. Proceed until no change has been accepted during an iteration of the algorithm.

Improvements to this algorithm lead us, for example, to a simulated annealing procedure or a multiple starting point procedure. However, as the number of parameters is very large (the number of pixels of the image), these improvements are very difficult to tailor and the convergence is very slow, especially for the simulated annealing version. Some results using these techniques will be presented for comparison with the genetic algorithm in the result section of the paper. (See Table 1 and (Otten and Ginneken, 1989).)

4. Inverse Fourier transform algorithm

This technique is derived from the classical *Projection Onto Convex Set* (*POCS*) algorithm which was first introduced by Levi and Stark (1983). This algorithm relies on imposing constraints and finding the optimal solution under these constraints. It is an iterative procedure where, during each iteration, the solution is projected onto each set of constraints at every iteration until a stop condition is met. Convergence is only ensured when the constraints define convex sets in the search space with a non-empty intersection.

Kotzer et al. (1993; 1994a; 1994b) extended the algorithm to non-convex sets with possibly empty intersection and proved the convergence. However, in the last case, the optimal solution does not lie in any of the constraint sets. In our example, a projection is therefore required to transform such an optimal (but useless) solution into a binary image X. This operation often leads to sub-optimal solutions when compared with solutions where the input binary nature of the image is taken into account during the iterative process itself. In our example, the two constraints are: the Fourier transform of the solution must match the given target, and, the solution must be binary. The algorithm is decomposed as follows:

1. Initialize an image X of size $N \times N$. Several initialization schemes are possible (random, inverse Fourier transform of the target T, etc.).

2. Take the Fourier transform: $Y = F(X)$. Impose the constraint in the Fourier plane. The target can be divided in two parts, a relevant part where we impose a constraint, the rest left free.
3. Take the inverse Fourier transform of the projection of Y and apply the other constraint in the space plane (take the real part and reduce the number of grey-levels)
4. Proceed until the image X is binary.

This solution has been applied to our problem and some results will be given in the result section of the paper (see Table 1).

5. Genetic algorithm

The *genetic algorithm* is a population-based iterative optimization method. Instead of just pushing iteratively a single candidate toward the optimal solution, it acts on a set of such solution candidates by simultaneously exploring several zones of the search space and by combining promising solution candidates, hopefully for the better.

We use a standard steady-state genetic algorithm (Davis, 1990) with an *elitist* selection strategy (the best candidate from one iteration is always kept to the next one notwithstanding that it might have been modified by mutation or crossover). The mutation operator is applied as usual on randomly chosen candidates and a single pixel of the candidate image is change.

As usual with genetic algorithms, deciding on a coding scheme is the critical part of the algorithm design. We choose the quadtree representation of a binary image as an efficient coding scheme but also to take into account the two-dimensional topological nature of an image. Quadtrees indeed, are renowned for their efficiency in coding binary images with the depths of the branches depending only on the homogeneity of the associated quadrants. They are also particularly well suited for combining by crossover topologically meaningful parts of solution candidates.

As a crossover operator we borrow the now classic tree crossover from John Koza's genetic programming (Koza, 1992). New solution candidates are created by swapping two arbitrarily chosen subtrees from two parent trees. This corresponds to exchanging quadrants of the same size but from potentially different image locations between the holograms associated with the parent solution candidates as can be seen in Fig. 1. Thus crossover acts on subtrees at the same level, possibly creating homogeneous branches. The maximal tree depth, of course, is limited by the image size.

A mutation operator has been introduced which arbitrarily changes the tree leaf values, and finally, a local optimization operator was added which tunes, by Direct Binary Search, a given number of best candidates after each iteration.

Table 1
Compared results of the different techniques for 32 × 32 images

Method	No. iterations	Uniformity	Efficiency
DBS	15360	97.1%	67.1%
SA	153600	91.9%	69.5%
IFTA	1000	89.1%	69.4%
GA	100000	98.8%	70.2%

Table 2
Compared results of the different techniques for 128 × 128 images

Method	No. iterations	Uniformity	Efficiency
DBS	150000	85.1%	60.1%
SA	200000	85.9%	63.5%
IFTA	1000	90.1%	70.4%
GA	150000	96.1%	71.5%

6. Results

The algorithm described in the previous section has been tested on two 32 × 32 target images, the first is composed of a 5 × 5 uniform square centered in the target T (see Fig. 2) and the second is composed of a 4 × 4 uniform square decentered in the target image $T1$ (see Fig. 2). The algorithm has also been tested on a 128 × 128 target image composed of a 5 × 5 uniform square centered in the target image. This target is the same as T but with a better resolution (i.e., more freedom degrees). This test is a very challenging one as the search space is huge.

The results are given in Tables 1 and 2 for the Direct Binary Search (DBS), Simulated Annealing (SA), the Inverse Fourier Transform (IFTA) and the genetic algorithm (GA) presented here. The results are average values for several experiments. We used two criteria to compare these different techniques: first, the efficiency of the reconstruction, i.e. the percentage of the input energy which is present in the relevant part of the target (the centered or decentered uniform square), second, the uniformity, which is a measure of the pre-

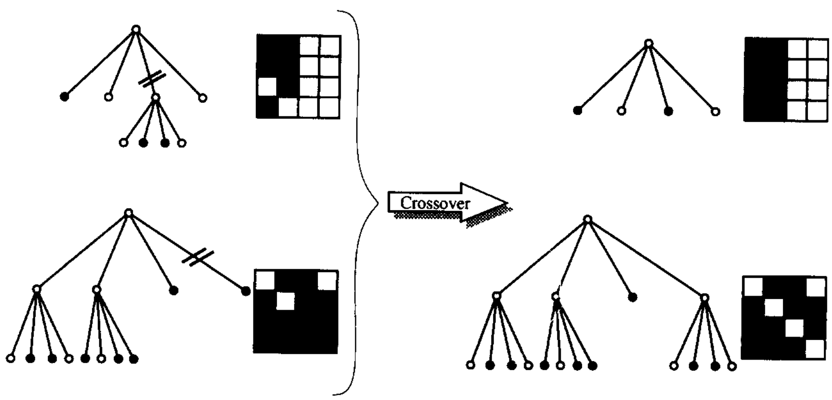

Fig. 1. Quadtree coding and the genetic programming tree crossover operator which swaps subtrees. The white (black) leaf nodes correspond to a quadrant of white (black) pixels.

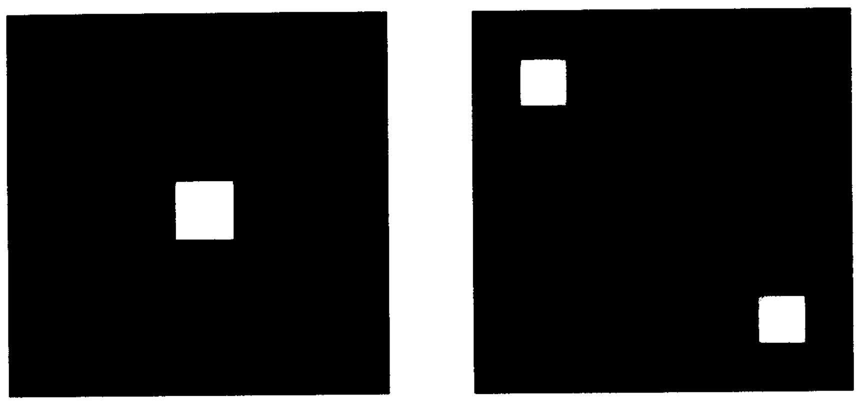

Fig. 2. Target images T and $T1$ respectively.

cision of the reconstruction. In our case efficiency and uniformity are defined as follows:

$$EFF(X) = \frac{\|F(X) \times T\|^2}{\|X\|^2}, \tag{3}$$

$$UNIF(X) = 1 - \frac{\max(\|F(X) - T\|^2) - \min(\|F(X) - T\|^2)}{\max(\|F(X) - T\|^2) + \min(\|F(X) - T\|^2)} \tag{4}$$

where *EFF* denotes the efficiency and *UNIF* the uniformity. The number of iterations corresponds to the number of calls to the Fast Fourier Transform (FFT).

Examples of 32×32 images for the target T and their associated reconstructions found by DBS and the GA are respectively shown in Figs. 3 and 4. The solutions in the space plane X are binary phase image, black and white pixels represent respectively π and 0 phase levels. The reconstruction is scaled on 256 grey-levels.

An example of the best solution found by GA in the other 32×32 case (target image $T1$) and its associated reconstruction is shown in Fig. 5.

An example of the best solution found by GA in the 128 × 128 case (target image T) and its associated reconstruction is shown in Fig. 6.

7. Conclusion

We have presented a new genetic algorithm coding scheme which takes into account the particular two-dimensional structure of an image. This new scheme has been tested for computer generated hologram design on a classical target example. The results have also been compared with those obtained with other techniques such as Hill Climbing, Simulated Annealing and Projection Onto Convex Set. The results on

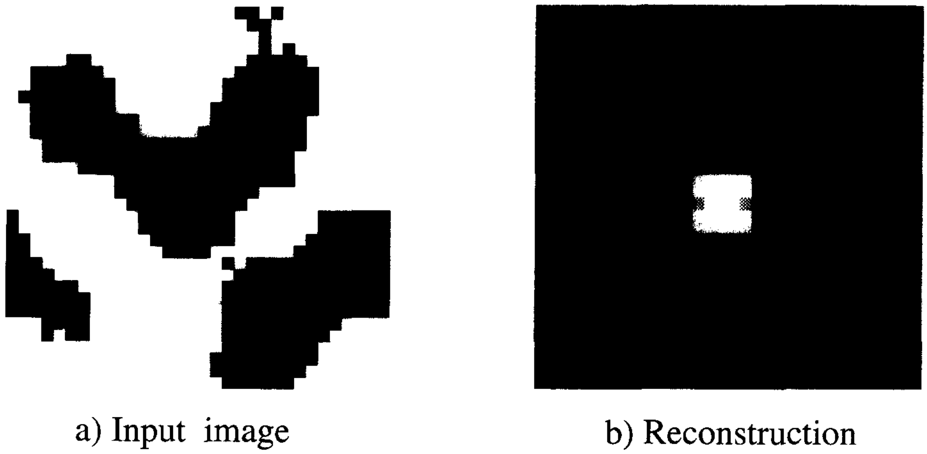

Fig. 3. An input image and its reconstruction by DBS method.

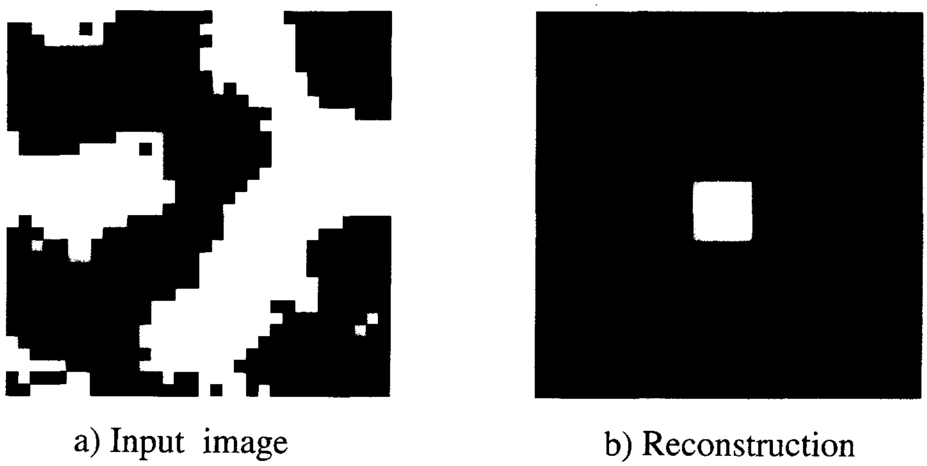

Fig. 4. An input image and its reconstruction by GA with quadtree coding method.

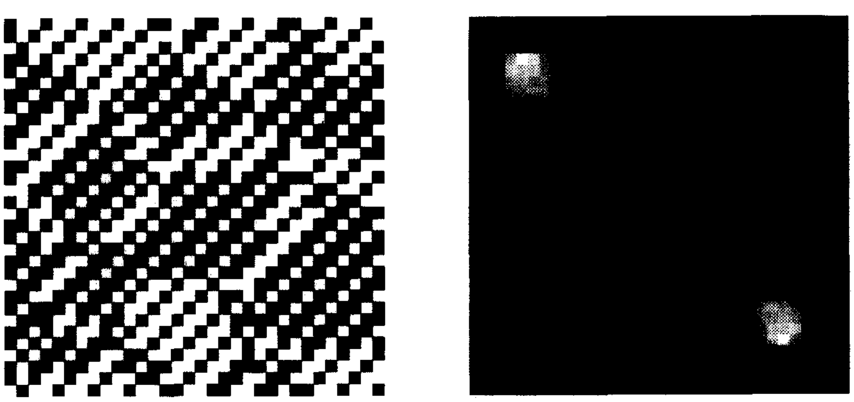

Fig. 5. An 32 × 32 input image and its reconstruction by GA with the quadtree coding method for target $T1$.

Fig. 6. An 128 × 128 input image and its reconstruction by GA with the quadtree coding method for target T.

this limited test are very encouraging leading to better performances than any other algorithm. However, further testing must be carried out to entirely prove the validity of our approach. More precisely, our approach should lead to very good results compared to DBS especially with larger images. Moreover, we can handle information of higher level such as classes in the case of image segmentation (natural GA feature) and also topological information with our new coding scheme. This could be very useful in various fields of image processing where optimization is involved.

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