Representation and Classification of 3-D Objects

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Abstract—This paper addresses the problem of generic object classification from three-dimensional depth or meshed data. First, surface patches are segmented on the basis of differential geometry and quadratic surface fitting. These are represented by a modified Gaussian image that includes the well-known shape index. Learning is an interactive process in which a human teacher indicates corresponding patches, but the formation of generic classes is unaided. Classification of unknown objects is based on the measurement of similarities between feature sets of the objects and the generic classes. The process is demonstrated on a group of three-dimensional (3-D) objects built from both CAD and laser-scanned depth data.

Index Terms—Classification, object recognition, 3-D vision.

I. INTRODUCTION

THE aim of this paper is to classify objects as belonging to generic model classes based on object shape. We consider the classification of objects from three-dimensional (3-D) data, perhaps derived from two-dimensional (2-D) images, but more commonly from a CAD design database or an active depth imaging system. The vast majority of previous systems, e.g., as reviewed in [5], have been directed toward object recognition rather than generic classification. These systems were usually highly constrained, e.g., limited to a given object class such as airplanes [8], using simple surface elements [7], and parametric object descriptions [3], [13], [19]. Many of these systems used local surface properties, of particular relevance are geometric histograms [2] and [16] and curvature signatures [1].

Our classification system, illustrated in Fig. 1, is different in many respects from the earlier work. It does not recognize an object as being identical or similar to another in an object database but rather classifies it as a member of an object class defined by previously learned exemplars. It employs only the two most important features needed for basic level recognition, i.e., the surface patches making up the bounding surface of the object and relations between them based on orientation. An object class contains a set of features, which it *must* have (minimum-feature-set), and another set of features, which it *may* have (maximal-feature-set). The elements of the minimum-feature-set may not be visible due to occlusion during data acquisition, but the more minimum-features that are present the more likely it is that the viewed object is a member of the class.

In the *training* phase, the operator presents objects to the system together with their class labels. The more consistent the

Fig. 1. Block diagram of the recognition process.

structure of the model class, the better the result expected from the system in a subsequent, automatic, *classification* phase. For example, if during training many totally different objects are placed in one single class, then the minimum-feature-set is empty. Thereafter, the classification of unknown objects within that class is not verifiable.

II. FEATURE EXTRACTION AND MODELLING

If geometric entities are labeled as one of a predefined group, e.g., cylindrical or spherical surfaces, this can be a barrier to higher order generalization. Avoiding strict type classification makes the recognition process more flexible, robust and invariant against shape variations of a certain scale (e.g., a cylindrical glass becomes parabolic). Our feature representation enables matching of features without labeling

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Fig. 2. Estimation of the surface normals and the first derivatives.

Fig. 3. Estimation of surface normals of the first derivative surface and the second derivatives.

by type, even if the feature to be recognized is only partially present. The scheme is based on unary and pairwise properties of surface patches, and achieves scale, position and orientation independence. Similar features have similar representations, and partial deletion of a feature causes partial deletion of the representation.

A. Segmentation of Three-Dimensional Meshed Objects

Our approach [5] is dependent on the base representation available from 3-D range sensors, i.e., a point cloud that can be represented as a triangular mesh. It has three main steps. The first step is the computation of the principal curvatures and labeling of the object's points. The mesh nodes are given in an arbitrary 3-D coordinate system as (x_i, y_i, z_i) triples, where i denotes the i -th node of the mesh. Mesh triangles are ordered lists of nodes. The surface normal at each node point is generated as the sum of the surrounding triangular facets' normals.

As shown in Fig. 2, the surface normal at a point of a surface uniquely defines the first partial derivatives in any given reference frame in any variable as $f_x(x, y) = -(n_x/n_z)$ and $f_y(x, y) = -(n_y/n_z)$. The computation of the second derivatives shown in Fig. 3 follows a similar path. One of the coordinates of the original node points is replaced by one of the partial derivatives. The reapplication of the above-defined process on the modified mesh yields two of the second partial derivatives. Replacing the node coordinates by the other first derivative, the other two partial derivatives can be generated. The replaced coordinate must be the one orthogonal to the parameter field. Although the surface curvatures are invariant to transformations of the parameter field, they are not entirely invariant to choosing another nonparallel parameter field. At the end of this stage, each node on the mesh has an associated label according

Fig. 4. Example of segmentation of a laser scanned teapot. The image on the left is the original depth data, shown as a shaded image, on the right are shown three patches of the nine that are extracted from the data.

to its mean and Gaussian curvature, defined as planar, cylindrical valley, cylindrical ridge, saddle pit, saddle ridge, spherical pit, and nonvalid points [3].

In the second stage, neighboring points with the same associated label are grouped to form surface patches. However, this description may be fragmented and is refined by a region-growing procedure (e.g., [3] and [10]). The result of this process is a set of surface patches, e.g., Fig. 4.

B. Surface Feature (SF)

1) *Quadratic Form:* After segmentation, each surface patch is represented by a general, ten-parameter, quadratic expression,

$$a_0 + [x \ y \ z] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + [x \ y \ z] \begin{bmatrix} a_4 & \frac{a_7}{2} & \frac{a_8}{2} \\ \frac{a_7}{2} & a_5 & \frac{a_9}{2} \\ \frac{a_8}{2} & \frac{a_9}{2} & a_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0. \quad (1)$$

The component matrix of the final bilinear term can be diagonalised by its modal matrix, $\underline{C}$ [12], [14]. Applying linear transformation of the coordinate system by the modal matrix to describe the same surface in a different frame, $f(\tilde{x}, \tilde{y}, \tilde{z}) = 0$ we have

$$a_0 + [\tilde{x} \ \tilde{y} \ \tilde{z}] \underline{C}^T \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \lambda_1 \tilde{x}^2 + \lambda_2 \tilde{y}^2 + \lambda_3 \tilde{z}^2 = 0 \quad (2)$$

where λ_1, λ_2 , and λ_3 are the eigenvalues of the original component matrix. Applying a translation of the coordinate system the linear components are eliminated. The final form of the original 10-parameter expression is one of those below, depending on the number of nonzero eigenvalues

$$\begin{aligned} \lambda_1 X^2 + \lambda_2 Y^2 + \lambda_3 Z^2 + \alpha &= 0, \\ \text{where } \alpha &= a_0 - \frac{\tilde{a}_1^2}{2\lambda_1} - \frac{\tilde{a}_2^2}{2\lambda_2} - \frac{\tilde{a}_3^2}{2\lambda_3} \\ \lambda_1 X^2 + \lambda_2 Y^2 + \tilde{a}_3 Z + \alpha &= 0, \\ \text{where } \alpha &= a_0 - \frac{\tilde{a}_1^2}{2\lambda_1} - \frac{\tilde{a}_2^2}{2\lambda_2} \\ \lambda_1 X^2 + \tilde{a}_2 Y + \tilde{a}_3 Z + \alpha &= 0, \\ \text{where } \alpha &= a_0 - \frac{\tilde{a}_1^2}{2\lambda_1} \quad \text{and} \\ \begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \tilde{a}_3 \end{bmatrix} &= \underline{C}^T \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \end{aligned} \quad (3)$$

The three expressions of (3) give three groups of quadrics, defined in Table I. However, the quadratic form is not useful for

TABLE I

THREE GROUPS OF QUADRIC SURFACES. THE TABLE SHOWS THE PARAMETER SIGNS FOR EACH SURFACE TYPE, AND IN THE LAST COLUMN THE GRADIENT PATTERNS OF QUADRATIC SURFACES WITH THE PRINCIPAL AXIS ALIGNED TO THE X AXIS. THE HORIZONTAL AND VERTICAL AXES REFER TO ν AND ϕ RESPECTIVELY. THE (0, 0) POINT IS IN THE MIDDLE OF THE PICTOGRAM

Group 1 surfaces				
Ellipsoid	$\alpha \neq 0$, all three roots are the same sign, which is opposite to α	$\alpha \geq 0, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, D \geq 0$	$\alpha \leq 0, \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, D \geq 0$	
Hyperboloid of one sheet	$\alpha \neq 0$, any two roots are of opposite sign, one is the same as α	$\alpha \geq 0, \lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, D \geq 0$	$\alpha \leq 0, \lambda_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, D \geq 0$	
		$\alpha \geq 0, \lambda_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, D \leq 0$	$\alpha \leq 0, \lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, D \leq 0$	
		$\alpha \geq 0, \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, D \leq 0$	$\alpha \leq 0, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, D \leq 0$	
Hyperboloid of two sheets	$\alpha \neq 0$, any two roots are of the same sign, one is opposite to α	$\alpha \geq 0, \lambda_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, D \leq 0$	$\alpha \leq 0, \lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, D \leq 0$	
		$\alpha \geq 0, \lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, D \geq 0$	$\alpha \leq 0, \lambda_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, D \geq 0$	
		$\alpha \geq 0, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, D \geq 0$	$\alpha \leq 0, \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, D \geq 0$	
Cone	$\alpha = 0$, any two of the roots are of the same sign, which is opposite to the third root	$\alpha = 0, \lambda_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, D = 0$	$\alpha = 0, \lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, D = 0$	
		$\alpha = 0, \lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, D = 0$	$\alpha = 0, \lambda_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, D = 0$	
		$\alpha = 0, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, D = 0$	$\alpha = 0, \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, D = 0$	
One point	$\alpha = 0$, all three roots are of the same sign	$\alpha = 0, \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, D = 0$	$\alpha = 0, \lambda_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, D = 0$	Not applicable
Group 2 surfaces				
Elliptic paraboloid	both roots are of the same sign	$(\tilde{\alpha}_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3) \text{ or } (\tilde{\alpha}_3 \lambda_1 \lambda_2 P_2 \geq 0)$ $(\tilde{\alpha}_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \leq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3)$ $(\tilde{\alpha}_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \geq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3) \text{ or } (\tilde{\alpha}_3 \lambda_1 \lambda_2 P_2 \leq 0)$ $(\tilde{\alpha}_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \leq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3)$		
Hyperbolic paraboloid	the two roots are opposite sign	$(\tilde{\alpha}_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \leq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3) \text{ or } (\tilde{\alpha}_3 \lambda_1 \lambda_2 P_2 \geq 0)$ $(\tilde{\alpha}_1 \leq 0, \lambda_2 \leq 0, \lambda_3 \geq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3)$ $(\tilde{\alpha}_1 \leq 0, \lambda_2 \geq 0, \lambda_3 \leq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3) \text{ or } (\tilde{\alpha}_3 \lambda_1 \lambda_2 P_2 \leq 0)$ $(\tilde{\alpha}_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \geq 0) \text{ or } (\tilde{\alpha}_2 \lambda_1 \lambda_3)$		
Elliptic cylinder	both roots are of the same sign	$(\tilde{\alpha}_1 = 0, \alpha \geq 0, \lambda_2 \leq 0, \lambda_3 \leq 0, P_1 = \pm \infty) \text{ or } (\tilde{\alpha}_2 \alpha \lambda_1 \lambda_3 P_1 = 0) \text{ or } (\tilde{\alpha}_3 \alpha \lambda_1 \lambda_2 P_2 = 0)$ $(\tilde{\alpha}_1 = 0, \alpha \leq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, P_1 = \pm \infty) \text{ or } (\tilde{\alpha}_2 \alpha \lambda_1 \lambda_3 P_1 = 0) \text{ or } (\tilde{\alpha}_3 \alpha \lambda_1 \lambda_2 P_2 = 0)$		
Hyperbolic cylinder	the two roots are of opposite sign	$(\tilde{\alpha}_1 = 0, \alpha \geq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, P_1 = \pm \infty) \text{ or } (\tilde{\alpha}_2 \alpha \lambda_1 \lambda_3 P_1 = 0) \text{ or } (\tilde{\alpha}_3 \alpha \lambda_1 \lambda_2 P_2 = 0)$ $(\tilde{\alpha}_1 = 0, \alpha \leq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, P_1 = \pm \infty) \text{ or } (\tilde{\alpha}_2 \alpha \lambda_1 \lambda_3 P_1 = 0) \text{ or } (\tilde{\alpha}_3 \alpha \lambda_1 \lambda_2 P_2 = 0)$		
		$(\tilde{\alpha}_1 = 0, \alpha \geq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, P_1 = \pm \infty) \text{ or } (\tilde{\alpha}_2 \alpha \lambda_1 \lambda_3 P_1 = 0) \text{ or } (\tilde{\alpha}_3 \alpha \lambda_1 \lambda_2 P_2 = 0)$ $(\tilde{\alpha}_1 = 0, \alpha \leq 0, \lambda_2 \geq 0, \lambda_3 \leq 0, P_1 = \pm \infty) \text{ or } (\tilde{\alpha}_2 \alpha \lambda_1 \lambda_3 P_1 = 0) \text{ or } (\tilde{\alpha}_3 \alpha \lambda_1 \lambda_2 P_2 = 0)$		
Group 3 surfaces				
Parabolic cylinder	One of $\tilde{\alpha}_1, \tilde{\alpha}_2$ and $\tilde{\alpha}_3 \neq 0$	$(\tilde{\alpha}_2 = 0, \tilde{\alpha}_3 \geq 0, P_1 = 0, P_2 \geq 0)$ $(\tilde{\alpha}_1 = 0, \tilde{\alpha}_3 \geq 0, P_1 = \pm \infty, P_2 \geq 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 P_2)$		
		$(\tilde{\alpha}_2 = 0, \tilde{\alpha}_3 \leq 0, P_1 = 0, P_2 \leq 0)$ $(\tilde{\alpha}_1 = 0, \tilde{\alpha}_3 \leq 0, P_1 = \pm \infty, P_2 \leq 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 P_2)$		
		$(\tilde{\alpha}_1 \geq 0, \tilde{\alpha}_3 \geq 0, P_2 \geq 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 = \text{const } P_2) \text{ or } (\tilde{\alpha}_2 \tilde{\alpha}_3 P_2)$ $(\tilde{\alpha}_1 \leq 0, \tilde{\alpha}_3 \geq 0, P_2 \geq 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 = \text{const } P_2) \text{ or } (\tilde{\alpha}_2 \tilde{\alpha}_3 P_2)$		
		$(\tilde{\alpha}_1 \geq 0, \tilde{\alpha}_3 \leq 0, P_2 \leq 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 = \text{const } P_2) \text{ or } (\tilde{\alpha}_2 \tilde{\alpha}_3 P_2)$ $(\tilde{\alpha}_1 \leq 0, \tilde{\alpha}_3 \leq 0, P_2 \leq 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 = \text{const } P_2) \text{ or } (\tilde{\alpha}_2 \tilde{\alpha}_3 P_2)$		
		$(\tilde{\alpha}_1 \leq 0, \tilde{\alpha}_3 = 0, P_2 = 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 = \text{const } P_2) \text{ or } (\tilde{\alpha}_2 \tilde{\alpha}_3 P_2)$ $(\tilde{\alpha}_1 \geq 0, \tilde{\alpha}_3 = 0, P_2 = 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 = \text{const } P_2) \text{ or } (\tilde{\alpha}_2 \tilde{\alpha}_3 P_2)$		
		$(\tilde{\alpha}_2 = 0, \tilde{\alpha}_3 = 0, P_1 = 0, P_2 = 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_3 P_1 P_2) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 P_2)$		
		$(\tilde{\alpha}_2 = 0, \tilde{\alpha}_3 = 0, P_1 = 0, P_2 = 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_3 P_1 P_2) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 P_2)$		
		Pair of parallel planes	Any two of $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3 = 0$	$(\tilde{\alpha}_2 = 0, \tilde{\alpha}_3 = 0, P_1 = 0, P_2 = 0) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_3 P_1 P_2) \text{ or } (\tilde{\alpha}_1 \tilde{\alpha}_2 P_1 P_2)$

comparison as small changes in the parameters can yield dramatic changes in the surface shape. Conversely, small changes in the shape or even rotations or translations can cause significant changes in the parameters of the quadratic expression. Therefore we develop a different surface representation to use for learning and matching surface descriptions.

2) *Modified Gaussian Image*: Since the patches generated by the segmentation do not contain inflections, they are concave, convex or saddle-shaped and the gradient distribution along the surface patch is continuous and monotonic. This is used to gen-

erate a modified Gaussian image (MGI). The two dimensions of the image are the ϑ and φ polar angles of the normal vectors, so that each surface shape has a characteristic gradient pattern. The stored value at each point in the gradient pattern is the local shape index [17], a function of the principal curvatures, κ_1 and κ_2 , that has been used in a number of previous systems, e.g., [11] and [18], and is independent of translation, rotation, and scaling

$$S_I = \frac{1}{\pi} \arctan \frac{\kappa_1 + \kappa_2}{\kappa_1 - \kappa_2}. \quad (4)$$

The shape index determines the local behavior of the surface and yields similar values for similar surfaces. The MGI is independent of scale and position, partial deletion of the object results in partial deletion of the pattern and it can also be shown that similar looking quadrics have similar patterns. The MGI is used in conjunction with the ratio of its surface area to the surface area of the entire object to form a surface feature (SF).

For each surface shown in Table I, we compute the expected form of the gradient pattern, that is the area of the Gaussian image (shaded dark) for which we can expect shape index values to be defined. To simplify the computations, the tangents of the two polar angles are used as parameters in addition to the two angles themselves.

$$\begin{aligned}\tan\vartheta &= \frac{f_y}{f_x} = p1 \quad -180^\circ < \vartheta \leq 180^\circ \\ \tan\varphi &= \frac{f_z}{\sqrt{f_x^2 + f_y^2}} = p2 \quad -90^\circ \leq \varphi \leq 90^\circ \\ \underline{n} &= (f_x, f_y, f_z).\end{aligned}\quad (5)$$

We show the procedure for surfaces belonging to the first group of Table I, but the computations for the other groups are similar and can be found in [6]. From the first (3), we have

$$\begin{aligned}\lambda_1 X^2 + \lambda_2 Y^2 + \lambda_3 Z^2 + \alpha &= 0 \quad p1 = \frac{2\lambda_2 Y}{2\lambda_1 X} \\ \underline{n} &= (2\lambda_1 X \quad 2\lambda_2 Y \quad 2\lambda_3 Z) \\ p2 &= \frac{2\lambda_3 Z}{\sqrt{(2\lambda_1 X)^2 + (2\lambda_2 Y)^2}}.\end{aligned}\quad (6)$$

Expressing Y and Z as functions of $p1$, $p2$, and X , then substituting them into the equation of the surface we can express X^2 . Similarly, we can express Y^2 and Z^2

$$\begin{aligned}\lambda_1 X^2 D + \alpha &= 0 \quad X^2 = \frac{-\alpha}{\lambda_1 D} \\ \lambda_2 Y^2 \frac{D}{p1^2 \frac{\lambda_1}{\lambda_2}} + \alpha &= 0 \quad Y^2 = \frac{-\alpha}{\lambda_2 D} \cdot p1^2 \frac{\lambda_1}{\lambda_2} \\ \lambda_3 Z^2 \frac{D}{(1 + p1^2)p2^2 \frac{\lambda_1}{\lambda_3}} + \alpha &= 0\end{aligned}\quad (7)$$

where

$$D = 1 + p1^2 \frac{\lambda_1}{\lambda_2} + p2^2 \frac{\lambda_1}{\lambda_3} + p1^2 p2^2 \frac{\lambda_1}{\lambda_3}.$$

The three equations give a valid solution if and only if $\alpha/\lambda_1 D \geq 0$, thus, $D \leq 0$ or $D \geq 0$ must hold depending on α and λ_1 . If $\alpha = 0$ then the only valid solution is $D = 0$. The shape of D depends on the sign of λ_1 , λ_2 , and λ_3 . If all three eigenvalues are of the same sign, D is always greater than zero. If one of the three eigenvalues is of different sign to the other two, then the shape of the $D = 0$ curve varies as shown in the entries for the hyperboloids and cone of Table I. This shows in the final column the characteristic gradient patterns of all the surface types.

The three groups of Table I are defined by their mathematical form, rather than their appearance. Surfaces in the same group may appear quite different, e.g., the hyperboloid of two sheets and the cone. Conversely, surfaces in different groups

Fig. 5. Mapping of points from an ellipsoid to a sphere.

may appear similar, especially when not complete, e.g., the elliptic paraboloid appears very similar to one part of the hyperboloid of two sheets or the ellipsoid. The lack of sharp divisions between the representations of surfaces is an essential feature of our approach. Different types of surface that appear similar when viewed partially generate a similar gradient pattern and shape index. Conversely, although the elliptic paraboloid gives a similar pattern to the ellipsoid in Table I, the attached shape indexes diverge at points further away from the origin, as the paraboloid becomes close to cylindrical at infinity. Therefore it is possible to discriminate when the surfaces are indeed quite dissimilar over a larger extent.

The gradient pattern is dependent on orientation. In the first group, except for the ellipsoid, there is one eigenvalue with opposite sign to the other two. The surfaces of the second group are linear in or lack one coordinate, whilst the parabolic cylinder, the only valid surface in the third group, is independent of one coordinate variable. We align the gradient pattern in a coordinate frame in which this unique axis is parallel to the X coordinate axis. To define the principal axis of the ellipsoid, it is mapped to a sphere as shown in Fig. 5. A plane is fitted to the mapped points. The axis of the quadratic closest to the normal vector of the fitted plane is chosen. Once the principal X axis is defined, the remaining degree of freedom can be eliminated by fixing the Y and Z coordinate axes to the remaining two axes of the quadratic.

3) *Computing the Gradient Pattern for Discrete Surfaces:* To generate the gradient pattern on a discrete surface, the normal vector and the two principal curvatures must be calculated. If we apply these computations directly to a sampled surface, it generates a gradient pattern at arbitrary points of the gradient space, making the comparison of two surfaces of different resolution difficult. Rather, the gradient space is uniformly sampled and represented in the ϑ - φ plane. The parametric form of the surface $Z = f(X, Y)$ is created, where X and Y are functions of $p1$ and $p2$. We chose to express Z as the function of the other two coordinates because the X -axis is the axis of the analytic surface, therefore this form avoids infinite-to-one mapping in the cases of objects containing parallel lines with the X -axis (e.g., cylinders). The differentiation necessary to compute the shape index is carried out in X and Y .

Figs. 6 and 7 show one example of the generation of the MGI from a single surface patch of a bottle. The pattern corresponds to an ellipsoid, although it is not complete over the ϑ - φ plane

Fig. 6. Partial ellipsoid defining the neck of a bottle.

Fig. 7. MGI of the SF in the ψ - φ plane. The tone of the shading is proportional to the shape index value of that point. The white shading represents a region without mapped points.

Fig. 8. Definition of the angle of coplanarity.

as it is not a complete surface. Although difficult to discern in a small monochrome image, the MGI is not uniformly shaded but has values corresponding to the local shape index. Since the gradient pattern of a plane is a single point and the shape index is not defined, there is no MGI defined for a planar surface. Hence, the final representation of a quadratic surface-feature is structured as

- 1) *quadratic/planar*;
- 2) *proportion of the surface area: area(patch)/area(object)*;
- 3) *MGI/nil*.

C. Orientation-Feature

An orientation-feature (OF) is a pairwise relationship between two SFs. The OF consists of two angles related to the *co-planarity* and *parallelism* of the orientation vectors of two surfaces. As before, we wish to avoid the generation of types so we adopt continuous measures.

1) *Co-Planarity*: Co-planarity, illustrated in Fig. 8, measures how close the orientation vectors of two surfaces are to being in one plane. It is an angle between two planes, P_1 and P_2 , defined by three space vectors, $\underline{v}_1$, $\underline{v}_2$, and $\underline{d}$, i.e., the two orientation vectors and the displacement vector, respectively

$$\text{coplanarity} = \arccos \left(\left| \frac{\underline{v}_1 \times \underline{d}}{\|\underline{v}_1\| \|\underline{d}\|} \cdot \frac{\underline{v}_2 \times \underline{d}}{\|\underline{v}_2\| \|\underline{d}\|} \right| \right). \quad (8)$$

Fig. 9. Definition of the angle of parallelism.

Since the directions of the orientation vectors are arbitrary, the co-planarity angle is represented in the $[0^\circ, 90^\circ]$ range.

2) *Parallelism*: Parallelism, illustrated in Fig. 9, is the angle between the two orientation vectors, $\underline{v}_1$ and $\underline{v}_2$, after they are transformed into the same plane. To transform the two vectors into a common plane one of the two vectors must be rotated around the displacement vector by the co-planarity angle. The rotated vector is labeled as $\underline{v}_2'$

$$\underline{v}_2' = \text{Rot}(\underline{v}_2, \underline{d}, \text{co-planarity})$$

$$\text{parallelism} = \arccos \left(\left| \frac{\underline{v}_1}{\|\underline{v}_1\|} \cdot \frac{\underline{v}_2'}{\|\underline{v}_2'\|} \right| \right). \quad (9)$$

Parallelism, also represented in the $[0^\circ, 90^\circ]$ range, is the angle between two intersecting lines in 2-D space and their direction is arbitrary. These definitions based on orientation vectors are unconventional and different from earlier work, e.g., [13], [15], and [20]. The final pairwise orientation feature representation is as follows.

- 1) Proportion of the surface area

$$(\text{Area}(\text{patch1}) + \text{Area}(\text{patch2})) / \text{Area}(\text{object}).$$

- 2) Parallelism: $\{\text{Parallelism} = X \mid X \in [0^\circ, 90^\circ]\}$.
- 3) Coplanarity: $\{\text{Coplanarity} = X \mid X \in [0^\circ, 90^\circ]\}$.

III. RECOGNITION AND CLASSIFICATION

The terms recognition and classification are often confused and used synonymously. In the following, the term recognition will be used to mean the classification of objects into groups and subgroups based on previous teaching, and not the recognition of an object as identical to one of the objects used in the teaching phase.

During class generation, objects of a class are introduced and the different manifestations of the same property of the class in different objects are extracted. The set of different object features, corresponding to the different manifestations of the same property of the class, make up a generalized representation of this class property, the *class feature*. This data structure includes all the *object features* corresponding to that class property in different members of the class, together with a learnt *tolerance* that relates to the variation of that class feature.

During recognition, all object features of the object to be recognized are matched against all class features of all known classes. Object features falling within the tolerance of a class feature give a high likelihood, whilst those outside the tolerance

represent a lower likelihood of the recognition of the object as an instance of the class.

A. Feature Matching

The categorization of objects is based on the extracted cues. There is no precedence and all object features are matched against all class features. Matching and comparison results in a number within the $[0, 1]$ interval (similarity) reflecting the likelihood of the match. Normalized metrics or weighting functions are used throughout and provide the generalization from object features to class features. The comparison of features is based on difference measures defined uniquely for each kind of feature.

1) *Surface Feature Measures*: There are three difference measures defined on two surface features (SF1, SF2).

SFM1: Average shape index difference

$$\text{SFM1} = \frac{1}{\|K_1 \cap K_2\|} \times \sum_{K_1 \cap K_2} |\text{SFM1}_{\text{MGI}}(\vartheta, \varphi) - \text{SFM2}_{\text{MGI}}(\vartheta, \varphi)|;$$

SFM2: Maximum overlap proportion

$$\text{SFM2} = 1 - \text{Max} \left(\frac{\|I\|}{\|K_1\|}, \frac{\|I\|}{\|K_2\|} \right);$$

SFM3: Size proportion difference

$$\text{SFM3} = |\text{SF1}_{\text{proportion}} - \text{SF2}_{\text{proportion}}|$$

where

$$I = \{(\vartheta, \varphi) : \vartheta \in [-180^\circ, 180^\circ], \varphi \in [-90^\circ, 90^\circ]\}$$

$$K_1 = \{(\vartheta, \varphi) \in I : -1 \leq \text{SF1}_{\text{MGI}}(\vartheta, \varphi) \leq 1\}$$

$$K_2 = \{(\vartheta, \varphi) \in I : -1 \leq \text{SF2}_{\text{MGI}}(\vartheta, \varphi) \leq 1\}$$

$\|I\|$, $\|K_1\|$, and $\|K_2\|$ are the cardinalities of sets I , K_1 , and K_2 , respectively.

The first measure is the average absolute difference between the shape indexes of the overlapping area of the two MGI's and reflects the similarity of shape of the two surfaces. The second measure is the highest ratio of the overlapping area of the two MGI's to the area of the MGI of each surface. This is necessary because, although the MGIs may be similar, two surfaces may not overlap to any great degree and may be wholly different for the most part. The third measure is the difference of the two proportions of the surface area of the surface patch to the entire object of which they are part. This ensures that similar looking but very differently sized parts of an object cannot be identified as similar, since the MGI representation is size invariant.

2) *Orientation Feature Measures*: There are three difference measures defined between two orientation features (OF1, OF2):

OFM1: Parallelism difference

$$\text{OFM1} = |\text{OF1}_{\text{parallelism}} - \text{OF2}_{\text{parallelism}}|.$$

OFM2: Coplanarity difference

$$\text{OFM2} = |\text{OF1}_{\text{coplanarity}} - \text{OF2}_{\text{coplanarity}}|.$$

OFM3: Size proportion difference

$$\text{OFM3} = |\text{OF1}_{\text{proportion}} - \text{OF2}_{\text{proportion}}|.$$

Fig. 10. Illustration of the binding of objects and class features in the database, and the minimum, maximum, and intermediate feature sets.

Since co-planarity and parallelism are angles between 0° and 90° , OFM1 and OFM2 must also be in the interval $[0^\circ, 90^\circ]$. OFM3 is represented in the $[0, 1]$ interval.

B. Teaching and Class Generation

There are three main steps to teaching and class generation. First, the representation of the new member of a class is inserted into the database. Second, the differences between features of the new and existing members of the class are computed and expressed in the form of a tolerance. Third, the generalization of the properties of the class is modified to reflect the contribution of the newly added class member to the definition of the class.

1) *Inserting a New Object Into Class*: On inserting the first object into a class the object features immediately become the class features, with a tolerance of zero. If a class is to be extended has at least one member, then the new object features have to be registered with the class features of the class so that the existing and new object features represent different manifestations of the same property. Fig. 10 illustrates the structure of classes and class features. The figure represents a class of three objects; object features (OFs) are arranged vertically, and corresponding features are grouped horizontally to define each class feature (CF) with an associated tolerance. If a feature of a new object is linked to an existing class feature, then the tolerance is recalculated. If a feature of a new object cannot be linked to an existing class feature, then it is introduced as a new class feature in the same way as a newly made class.

2) *Computing the Tolerance on Class Features*: A class feature is the generalization of the corresponding object features. When a new object is inserted into a class, the generalization of the properties of the class is modified. The tolerance indicates the diversity of the manifestations of the object features building up the particular class feature, and is related to the minimum and the maximum differences between the corresponding object features. Since any object feature of an object to be recognized matching an object feature of the class feature exactly must fall always within the boundaries of the tolerance, the minimum dif-

Fig. 11. (a) Conversion of a typical difference measure to likelihood. (b) Conversion of SFM difference measure to likelihood.

ference is always zero. To calculate the maximum difference between the object features of a class feature all the object features must be compared. This yields a number of difference measures. The maximum is stored as the upper tolerance bound.

C. Classification of Objects

The aim of the classification process is to determine whether the object to be recognized is a manifestation of the objects already in the class. This process consists of five main steps. First, all object features of the object to be recognized are compared to all class features of a class. Second, the most similar class feature to each object feature is determined. Third, indicators are created to signal different aspects of the object to be recognized as being a member of the class. Fourth, the indicators are combined into an overall likelihood. Fifth, the class with the highest likelihood value is selected.

1) *Class Feature to Object Feature Comparison:* In classification, the likelihood of two features matching depends on the definition of the class to which they belong. If the objects of a class are identical in one particular property then another object different in this property is not likely to be an instance of this class. If the members of the class vary in a property then an object not matching closely the class members in this property can still be a candidate for membership of the class.

In comparing an object to one contained in the database of Fig. 10, a *similarity measure* S is computed in the interval $[0, 1]$. This is a function of both the class feature tolerance and the object feature differences defined in Section III.A. If the object feature difference falls inside the tolerance of the respective feature of the class then the similarity measure is 1. This decreases linearly with the distance from the upper edge of the tolerance, as illustrated in Fig. 11. The zero similarity point varies from measure to measure. In most cases this value is the maximum value of the corresponding difference measure. In the case of SFM1, the shape index domain can be divided into eight main sections, defined by the sign change of the mean and Gaussian curvatures. These are fundamentally different, therefore no surfaces further from 1/8th of the scale should be registered as the same with any likelihood. Therefore the zero likelihood point is set within a 0.25 shape index value distance from the middle of the tolerance.

The multiple similarities are combined to give a single value reflecting the combined *likelihood* of the two features being different manifestations of the same class property. In the case of a surface-feature the combined similarity is the product of the independent similarities. In the case of an orientation-feature, the similarity measures of OFM1 and OFM2 are averaged (as they

are strongly related), then the result is multiplied by the likelihood evaluated from OFM3.

$$L_{\text{surface_feature_match}} = S_{\text{SFM1}} \cdot S_{\text{SFM2}} \cdot S_{\text{SFM3}}$$

$$L_{\text{orientation_feature_match}} = \frac{S_{\text{OFM1}} \cdot S_{\text{OFM2}}}{2} \cdot S_{\text{OFM3}}.$$

The comparison of an object with a class requires comparison of all object features with all class features. The object feature with the highest likelihood is assigned first to the corresponding class feature then removed from the ordered list of any other class feature.

2) *Computing the Likelihood of Class Membership:* The previous step yields the likelihood that each feature of the object to be recognized is a manifestation of the class property represented by the class feature. The most important class features are present in all objects of the class, whilst the least important features appear in only one object. The features present in all the members of a class form the *minimum-feature-set*. The set of all the class features of a class together is called the *maximum-feature-set*. The set of those class features which appear in more than one but not all objects of the class form *intermediate-feature-sets* with various cardinalities.

The final likelihood of an object being recognized as a member of an existing class is based on *indicators* that account for the likelihoods and the cardinality of the feature set to which the class feature belongs. The first indicator, I_j , where $j = 1$, is the weighted average of the likelihoods of all the class features that contain at least one object feature where j is the cardinality of the feature set, $0 < j < N_o$ (number of objects in the class). There are as many indicators as there are objects in the class.

$$I_j = \frac{\sum_{i=1}^N C_{CFi} \cdot L_{CFi}}{\sum_{i=1}^N C_{CFi}} \quad \forall C_{CFi} \geq j$$

Thus, if $j = 1$ then $I_j = I_I = I_{\text{maximum-feature-set}}$, whilst if $j = N_o$ then $I_j = I_M = I_{\text{minimum-feature-set}}$. There is an additional indicator, I_A , the average likelihood of all matching class features, i.e., class features with zero likelihood are excluded.

$$I_A = \frac{\sum_{i=1}^N C_{CFi} \cdot L_{CFi}}{\sum_{i=1}^N C_{CFi}} \quad \forall L_{CFi} > 0.$$

IV. EXPERIMENTAL EVALUATION

We present a library of 21, 3-D-meshed objects in Fig. 12; those in the first three rows, mug (b) and bottles (e) and (f) are synthetic, the others meshed from real data. The experiments show the effects of different allocations of objects to classes

Fig. 12. Objects employed in the experiments.

TABLE II
INDICATORS OF FINAL LIKELIHOOD OF THE OBJECTS BELONGING TO THE WEDGE CLASS. THE OBJECTS IN THE SHADED LINES ARE USED DURING THE TEACHING PHASE

	I_1	I_2	$I_3=I_M$	I_A	$I_M \cdot I_A$
Mug a.)	0.41	0.33	0.33	0.59	0.20
Mug b.)	0.13	0.11	0.11	0.64	0.07
Mug c.)	0.13	0.11	0.11	0.65	0.08
Mug d.)	0.50	0.41	0.41	0.72	0.29
Mug e.)	0.36	0.26	0.26	0.53	0.14
Mug f.)	0.36	0.26	0.26	0.52	0.13
Bottle a.)	0.11	0.10	0.10	0.62	0.06
Bottle b.)	0.26	0.24	0.24	0.53	0.13
Bottle c.)	0.15	0.15	0.15	0.49	0.07
Bottle d.)	0.21	0.24	0.24	0.55	0.13
Bottle e.)	0.01	0.00	0.00	0.31	0.00
Bottle f.)	0.29	0.27	0.27	0.59	0.16
Cube	0.62	0.75	0.75	0.91	0.68
Cut-block	0.72	0.64	0.64	0.90	0.58
10-sided-block	0.86	0.87	0.87	0.86	0.75
Pyramid	0.60	1.00	1.00	1.00	1.00
Slope	0.60	1.00	1.00	1.00	1.00
Wedge	0.60	1.00	1.00	1.00	0.99
Trapezoid	0.59	0.61	0.61	0.91	0.56
Half-house	0.60	0.66	0.66	0.93	0.61
Asymmetric-slope	0.58	0.98	0.98	0.97	0.96

during teaching, the discriminatory power of the similarity measures and indicators that include set cardinality, and evaluate the accuracy of classification of unknown objects into pre-taught classes. In general, the indicators can be used in different ways according to the specific task. In these examples, the final likelihood of an object belonging to a class is the product of the last two indicators, of the minimum-feature-set and of the matching features. The former is used as it represents the core of the class and the latter as it represents the similarity of the matching features. In Fig. 12 and Tables II–VI, we summarize the results of building and comparing the 21 objects to a set of classes defined as {box, wedge, mug, bottle}. Alternative classifications are valid, e.g., the last two objects of the bottle class could be taught as “vases.”

TABLE III
INDICATORS OF FINAL LIKELIHOOD OF THE OBJECTS BELONGING TO THE BOX CLASS. THE OBJECTS IN THE SHADED LINES ARE USED DURING THE TEACHING PHASE

	I_1	I_2	$I_3=I_M$	I_A	$I_M \cdot I_A$
Mug a.)	0.40	0.33	0.33	0.57	0.19
Mug b.)	0.09	0.11	0.11	0.62	0.07
Mug c.)	0.11	0.16	0.16	0.66	0.10
Mug d.)	0.52	0.44	0.44	0.64	0.28
Mug e.)	0.37	0.31	0.31	0.53	0.16
Mug f.)	0.30	0.24	0.24	0.55	0.13
Bottle a.)	0.10	0.14	0.14	0.86	0.12
Bottle b.)	0.17	0.14	0.14	0.80	0.11
Bottle c.)	0.12	0.13	0.13	0.77	0.10
Bottle d.)	0.13	0.13	0.13	0.80	0.10
Bottle e.)	0.01	0.00	0.00	0.31	0.00
Bottle f.)	0.17	0.14	0.14	0.81	0.11
Cube	0.61	1.00	1.00	1.00	1.00
Cut-block	0.65	0.94	0.94	0.99	0.94
10-sided-block	0.93	0.99	0.99	1.00	0.99
Pyramid	0.35	0.56	0.56	0.90	0.50
Slope	0.37	0.59	0.59	0.94	0.56
Wedge	0.31	0.45	0.45	0.91	0.41
Trapezoid	0.51	0.808	0.80	0.93	0.75
Half-house	0.49	0.76	0.76	0.93	0.71
Asymmetric-slope	0.34	0.53	0.53	0.91	0.48

TABLE IV
INDICATORS OF FINAL LIKELIHOOD OF THE OBJECTS BELONGING TO THE MUG CLASS. THE OBJECTS IN THE SHADED LINES ARE USED DURING THE TEACHING PHASE

	I_1	I_2	$I_3=I_M$	I_A	$I_M \cdot I_A$
Mug a.)	0.71	1.00	1.00	1.00	1.00
Mug b.)	0.23	0.52	0.91	0.91	0.82
Mug c.)	0.25	0.57	0.99	0.99	0.98
Mug d.)	0.64	0.64	0.56	0.86	0.48
Mug e.)	0.72	1.00	1.00	1.00	1.00
Mug f.)	0.49	0.57	0.60	0.92	0.55
Bottle a.)	0.13	0.22	0.19	0.86	0.16
Bottle b.)	0.24	0.35	0.32	0.86	0.27
Bottle c.)	0.2	0.36	0.31	0.87	0.26
Bottle d.)	0.18	0.25	0.32	0.85	0.27
Bottle e.)	0.03	0.07	0.09	0.47	0.04
Bottle f.)	0.25	0.41	0.42	0.83	0.35
Cube	0.17	0.31	0.33	0.73	0.24
Cut-block	0.22	0.37	0.44	0.74	0.32
10-sided-block	0.34	0.37	0.36	0.68	0.24
Pyramid	0.14	0.25	0.35	0.77	0.27
Slope	0.15	0.29	0.36	0.8	0.29
Wedge	0.15	0.29	0.41	0.76	0.31
Trapezoid	0.18	0.30	0.36	0.79	0.28
Half-house	0.18	0.32	0.35	0.78	0.28
Asymmetric-slope	0.15	0.29	0.36	0.79	0.28

TABLE V
INDICATORS OF FINAL LIKELIHOOD OF THE OBJECTS BELONGING TO THE BOTTLE CLASS. THE OBJECTS IN THE SHADED LINES ARE USED DURING THE TEACHING PHASE. AS ONLY TWO OBJECTS ARE USED TO CONSTRUCT THE CLASS, THE MIDDLE FEATURE SET COLUMN IS EMPTY

	I_1	$I_2=I_M$	I_A	$I_M \cdot I_A$
Mug a.)	0.91	0.91	0.91	0.82
Mug b.)	0.35	0.41	0.58	0.23
Mug c.)	0.35	0.41	0.55	0.22
Mug d.)	0.85	0.87	0.88	0.76
Mug e.)	0.91	0.93	0.91	0.85
Mug f.)	0.90	0.93	0.93	0.87
Bottle a.)	0.80	1.00	1.00	1.00
Bottle b.)	0.83	0.85	0.87	0.74
Bottle c.)	1.00	1.00	1.00	1.00
Bottle d.)	0.74	0.75	0.77	0.57
Bottle e.)	0.10	0.13	0.43	0.05
Bottle f.)	0.77	0.80	0.88	0.70
Cube	0.36	0.38	0.57	0.22
Cut-block	0.41	0.42	0.64	0.27
10-sided-block	0.44	0.40	0.69	0.27
Pyramid	0.34	0.35	0.54	0.19
Slope	0.33	0.33	0.51	0.17
Wedge	0.28	0.28	0.43	0.12
Trapezoid	0.38	0.38	0.59	0.22
Half-house	0.36	0.36	0.57	0.21
Asymmetric-slope	0.31	0.32	0.48	0.15

Tables II–V show the objects used in this experiment during the teaching phase as shaded rows, and the results of classification of all objects with respect to each class, including the original objects used for teaching and additional unseen objects. The columns show the indicator values obtained and the final likelihood value. There are two objects misclassified in Table VI, i.e.,

TABLE VI
FINAL LIKELIHOOD OF THE OBJECTS BELONGING TO THE MUG, BOTTLE, BOX OR THE WEDGE CLASS. THE SHADED CELLS INDICATE THE MOST LIKELY CLASS TO WHICH THE OBJECTS BELONG. THE UNDERLINED VALUES INDICATE INCORRECT CLASSIFICATION

	Mug	Bottle	Box	Wedge
Mug a.)	1.00	0.82	0.19	0.20
Mug b.)	0.82	0.23	0.07	0.07
Mug c.)	0.98	0.22	0.10	0.08
Mug d.)	0.48	0.76	0.28	0.29
Mug e.)	1.00	0.85	0.16	0.14
Mug f.)	0.55	0.87	0.13	0.13
Bottle a.)	0.16	1.00	0.12	0.06
Bottle b.)	0.27	0.74	0.11	0.13
Bottle c.)	0.26	1.00	0.10	0.07
Bottle d.)	0.27	0.57	0.10	0.13
Bottle e.)	0.04	0.05	0.00	0.00
Bottle f.)	0.35	0.70	0.11	0.16
Cube	0.24	0.22	1.00	0.68
Cut-block	0.32	0.27	0.94	0.58
10-sided-block	0.24	0.27	0.99	0.75
Pyramid	0.27	0.19	0.50	1.00
Slope	0.29	0.17	0.56	1.00
Wedge	0.31	0.12	0.41	0.99
Trapezoid	0.28	0.22	0.75	0.56
Half-house	0.28	0.21	0.71	0.61
Asymmetric-slope	0.28	0.15	0.48	0.96

mugs (d) and (f). This is due to the similarity of the “neck” structure of the three adjacent coaxial surfaces of the bottle class and these two mugs. Although the “neck” of mug (f), for example, is at its base, it is pertinent to recall that our classification is orientation independent. A human observer might also emphasise the importance of the “handle,” suggesting greater weighting for this feature, but again we stress that the teaching phase is automatic except for identification of corresponding features, and no particular precedence is enforced here. As expected, the comparison of objects containing planar faces only to the mug and bottle classes yields low likelihood values. Though most of the planar object surface features of the objects of the wedge and box classes do not match any class surface features of the mug and the bottle classes, there are matches amongst the orientation features, increasing the likelihood of the object being a manifestation of the class. This is also true for the classes contacting objects with only planar faces and the bottles and mugs. The results for the trapezoid, half-house, and asymmetric slope objects are informative, as these were designed to have strong apparent similarities to both the wedge and box classes. This shows in the classification and respective final likelihood values summarized in Table VI.

It is also instructive to compare the teapot of Fig. 4 to the database of Table VI. If the mug class is defined by mugs (a), (c), and (e) the set of likelihood values is $\{0.32(\text{mug}), 0.67(\text{bottle}), 0.21(\text{box}), 0.44(\text{wedge})\}$. However if the mugs (b), (c), and (d) are used, then the similarity to the mug class increases to 0.59. In this case the key feature is the bulbous body of mug (d). If mug (d) is not used then the cylindrical shape of the cups forms a distinct contrast to the teapot and the bottles are considered much more likely in the absence of a teapot class. If it is used, then the class of mugs is a much likelier classification.

V. CONCLUSION

We have presented an approach to classification of 3-D objects input as a cloud of points or mesh according to type, rather than as an instance of a specific object. The approach is based

on segmentation of the bounding surface of objects into patches that can be described by shape properties, notably a modified Gaussian image, and orientation properties, co-planarity and parallelism measures, though these are not conventionally defined. The approach is scale-, orientation-, and translation-independent.

Learning and class generation are key components of the work. The system learns by example, guided only by the teacher in the allocation of corresponding surface patches. Hence, different classes and different common features can be constructed during the teaching phases. During classification, our approach makes use of similarity measures based on minimum, intermediate and maximum feature sets to arrive at a final likelihood of membership of an unknown object of a class.

We have shown here the operation of our approach on an exemplary set of objects that are constructed from planar and curved surfaces. Fuller experimental evaluation can be found in [6]. The system is open to extension without changing the conceptual structure. For example, additional features based on topology or connection, or nonshape-based features such as color or texture could be added to the system together with their class generation method.

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